Teachers' Edition

Investigating School Mathematics

Extending the Ideas

Investigating School Ma

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ADDISON-WESLEY PUBLISHING COMPANY

Menlo Park, California · Reading, Massachusetts · London · Don Mills, Ontario

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ISBN 0-201-09538-6

CDEFGHIJKL—WC—787

Estimating Measurements

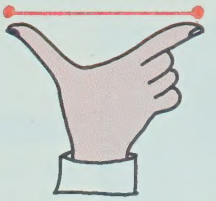
In each part below, use the unit shown. First, make an estimate (clever guess) about the number of units needed. Then measure to check your guess.

1. Unit:  your pace

How far is it across your room?

A Your estimate: Answers will vary.

B Measured distance: Answers will vary.

2. Unit:  your thumb-finger opening

How wide is the chalkboard in your classroom?

A Your estimate: Answers will vary.

B Measured length: Answers will vary.

3. Unit:  your shoe length

How far is it from your chair to the pencil sharpener?

A Your estimate: Answers will vary.

B Measured distance: Answers will vary.

4. Unit:  your hand width

How wide is a desk in your classroom?

A Your estimate: Answers will vary.

B Measured width: Answers will vary.

5. Unit:  your thumb width

How long is your book?

A Your estimate: Answers will vary.

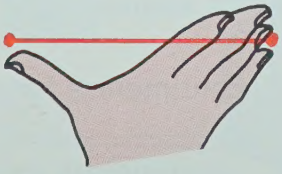
B Measured length: Answers will vary.

6. Unit:  your arm span

How long is your classroom?

A Your estimate: Answers will vary.

B Measured length: Answers will vary.

7. Unit:  your span

How wide is a window in your classroom?

A Your estimate: Answers will vary.

B Measured width: Answers will vary.

8. Unit:  your little finger width

How far is it from your elbow to your finger tip?

A Your estimate: Answers will vary.

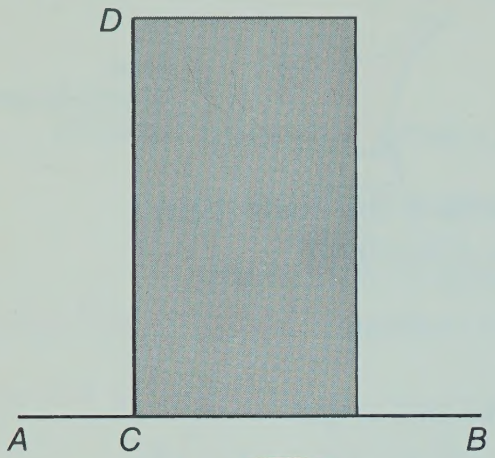
B Measured distance: Answers will vary.

The word estimate may be new and require some explanation.

Using a Centimeter Ruler

First circle longer, shorter, or same.
Then measure to find out if you were right.

1.

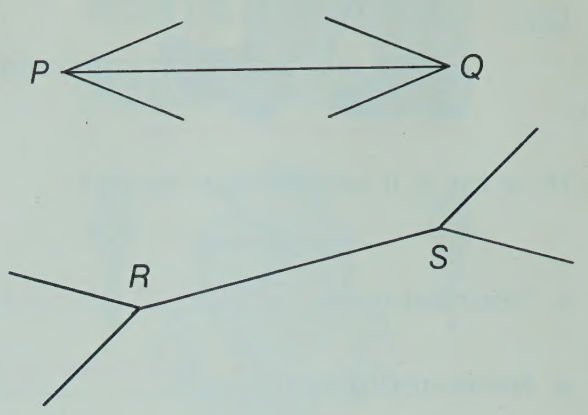


Is segment *AB* **longer**, **shorter**, or the **same** as segment *CD*?

$\overline{AB}$ is 6 centimeters long.

$\overline{CD}$ is 5 centimeters long.

2.

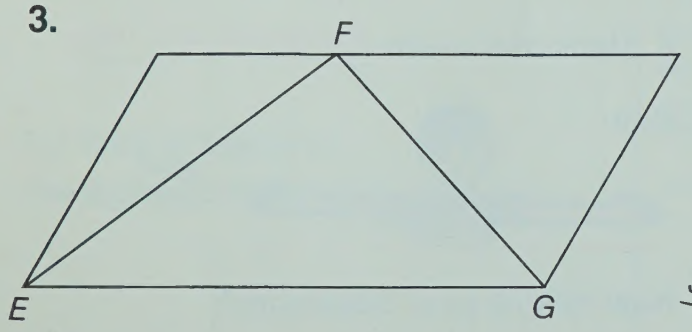


Is segment *PQ* **longer**, **shorter**, or the **same** as segment *RS*?

$\overline{PQ}$ is 5 centimeters long.

$\overline{RS}$ is 4 centimeters long.

3.

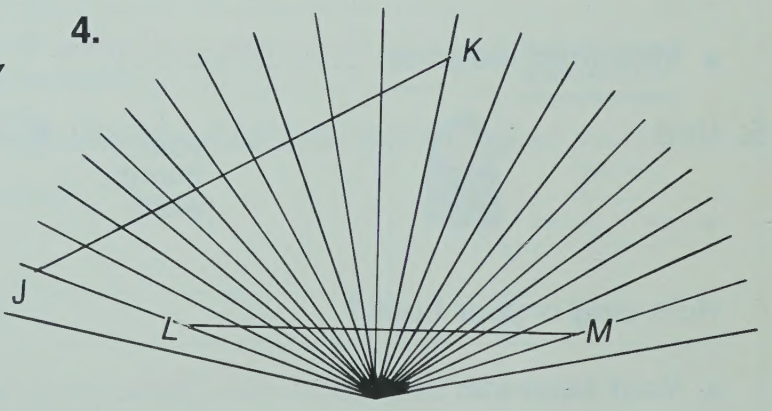


Is segment *EF* **longer**, **shorter**, or the **same** as segment *FG*?

$\overline{EF}$ is 5 centimeters long.

$\overline{FG}$ is 4 centimeters long.

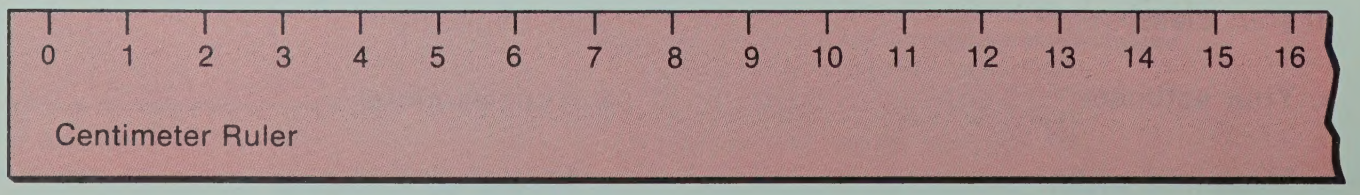
4.



Is segment *JK* **longer**, **shorter**, or the **same** as segment *LM*?

$\overline{JK}$ is 6 centimeters long.

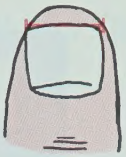
$\overline{LM}$ is 5 centimeters long.



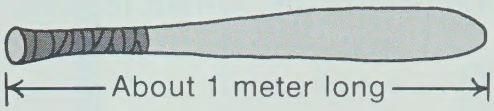
If children do not have a centimeter ruler have them cut out the one at the bottom of the page. Explain that $\overline{AB}$ is read "segment *AB*."

Units in the Metric System

A **centimeter** unit is about this long.



A **meter** unit is 100 centimeters long.
It is about as long as a large baseball bat.



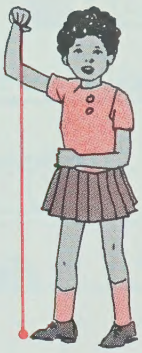
Estimate the following lengths. Then use a centimeter ruler or a meter stick to check your estimates. Answers will vary.

1. Cut a piece of string and put it around your waist.

A Estimate the distance around your waist in centimeters. _____

B Measured distance: _____ centimeters.

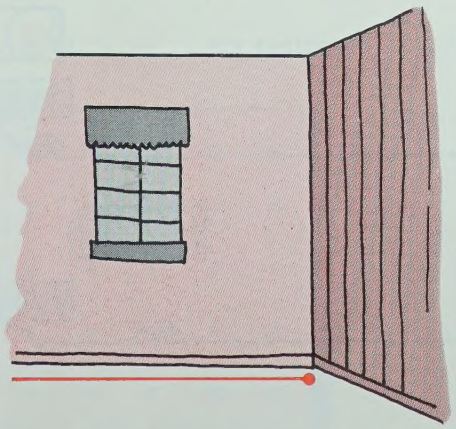
2. Cut a piece of string that is exactly your height.



A Estimate your height in centimeters. _____

B Measured height: _____ centimeters.

3.



A Estimate the length of your room in meters. _____

B Measured length: _____ meters.

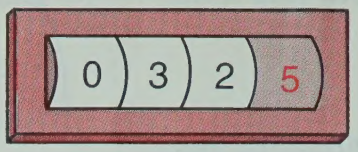
4. Draw a picture of something you would like to measure. Then fill in the blanks below.

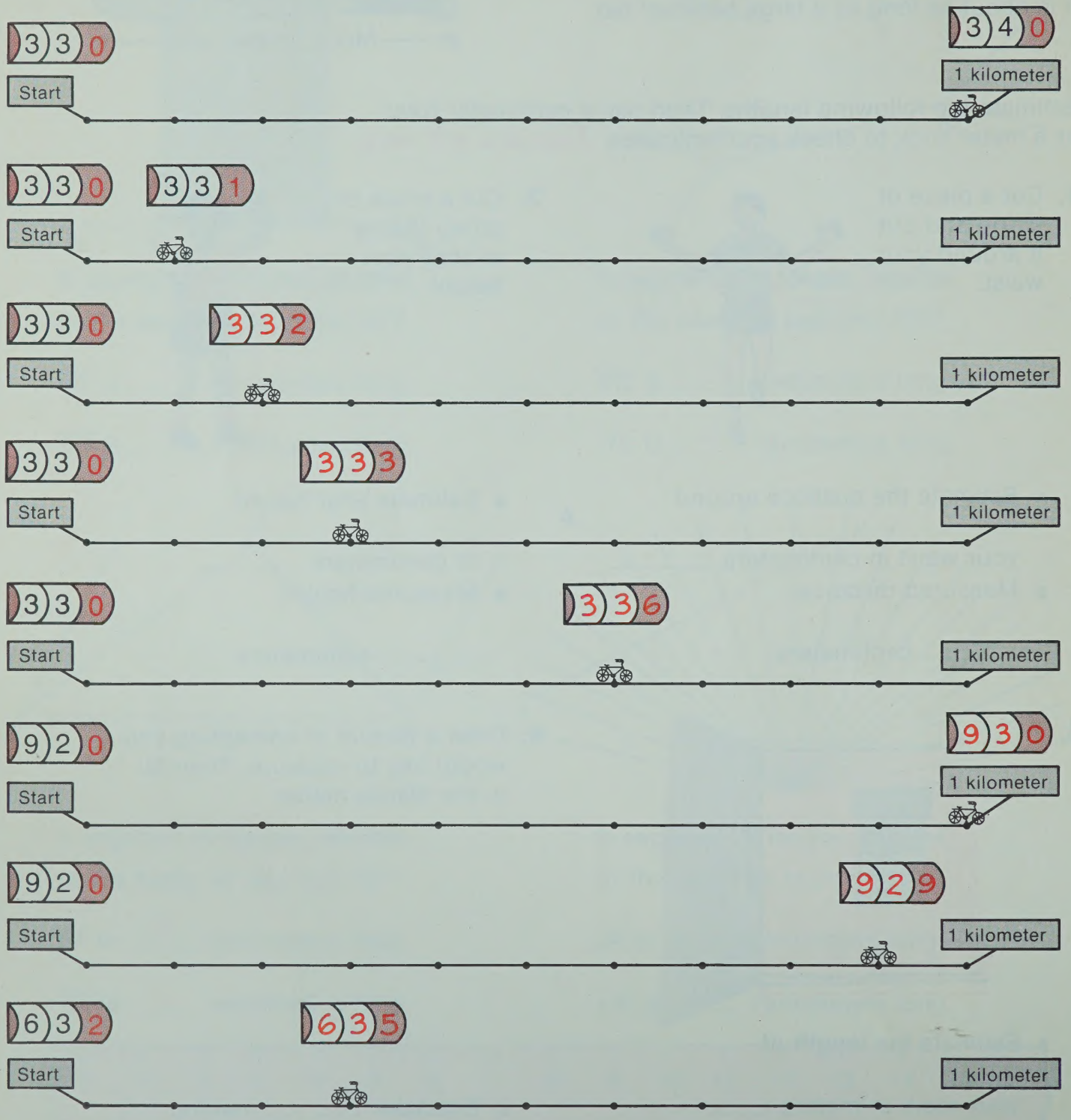
A Estimate: _____ meters.

B Measurement: _____ meters.

Using Tenths in Measurement

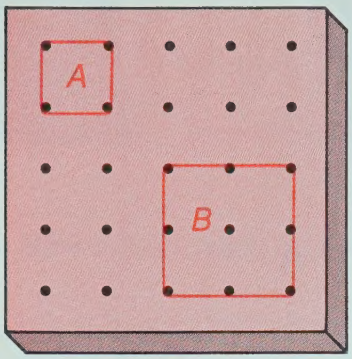
Eric had an **odometer** on his bicycle. The red numeral told how many extra tenths of a kilometer he had traveled. Write the numerals so that each **odometer** shows the correct reading.





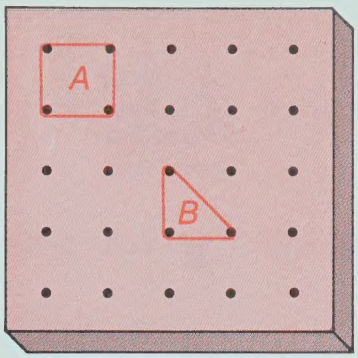
Area on the Geoboard

Find the area of each figure. The area of this figure is 1 square unit. 

1. 

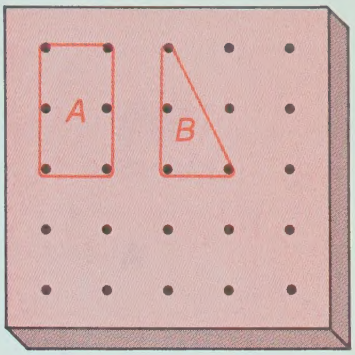
Area A: 1 sq units

Area B: 4 sq units

2. 

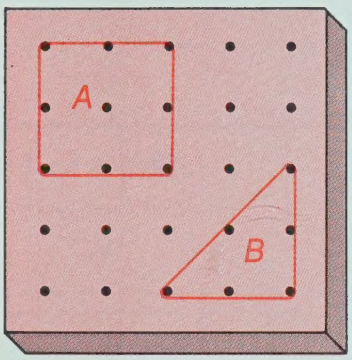
Area A: 1 sq units

Area B: $\frac{1}{2}$ sq units

3. 

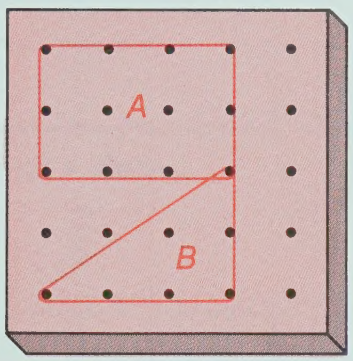
Area A: 2 sq units

Area B: 1 sq units

4. 

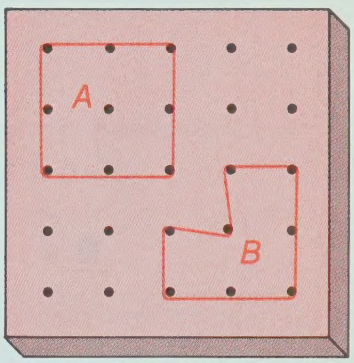
Area A: 4 sq units

Area B: 2 sq units

5. 

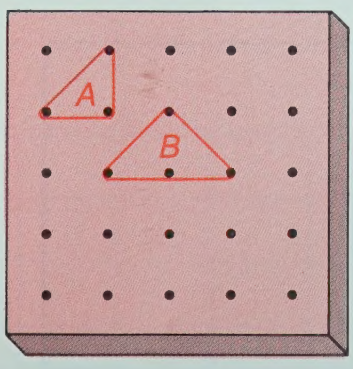
Area A: 6 sq units

Area B: 3 sq units

6. 

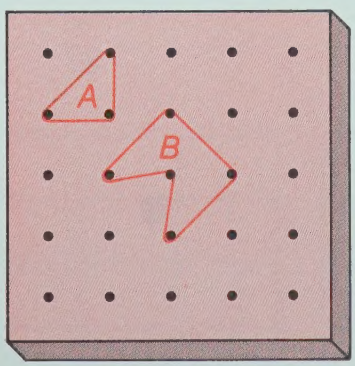
Area A: 4 sq units

Area B: 3 sq units

7. 

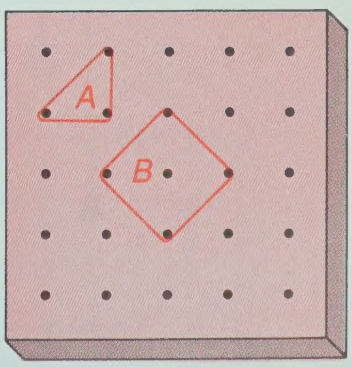
Area A: $\frac{1}{2}$ sq units

Area B: 1 sq units

8. 

Area A: $\frac{1}{2}$ sq units

Area B: $1\frac{1}{2}$ sq units

9. 

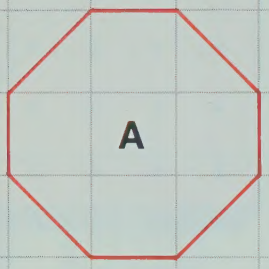
Area A: $\frac{1}{2}$ sq units

Area B: 2 sq units

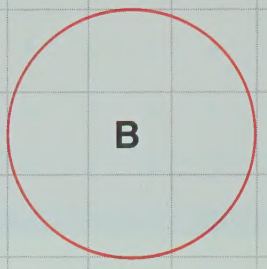
For some children this may be their first exposure to a geoboard. Actually using the geoboard to form the figures will prove helpful for most children. The "trick" here is to figure halves of various squares and rectangles.

Estimating Area

Find the area of the A figures. You will not be able to find the area of the B figures exactly. Estimate the area of the B figures.

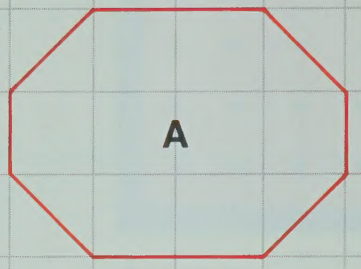


Area: 7 sq units

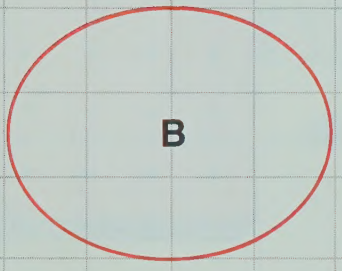


Estimated Area:

7 sq units

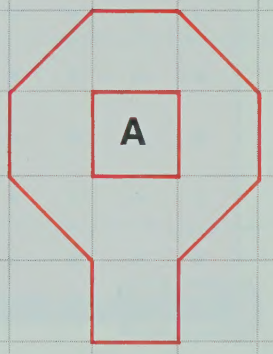


Area: 10 sq units



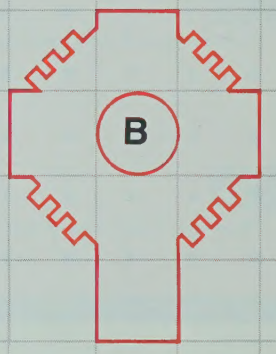
Estimated Area:

10 sq units



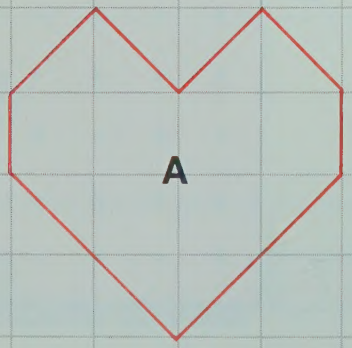
Area:

7 sq units

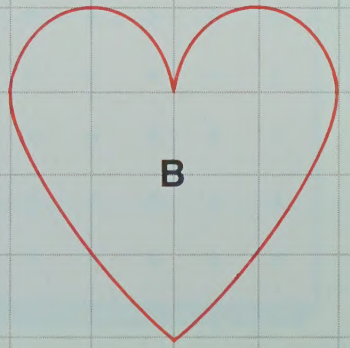


Estimated Area:

7 sq units

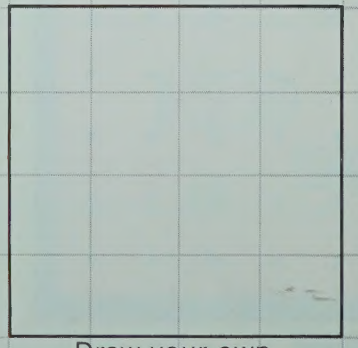


Area: 10 sq units



Estimated Area:

10 sq units



Draw your own.

Estimated Area:

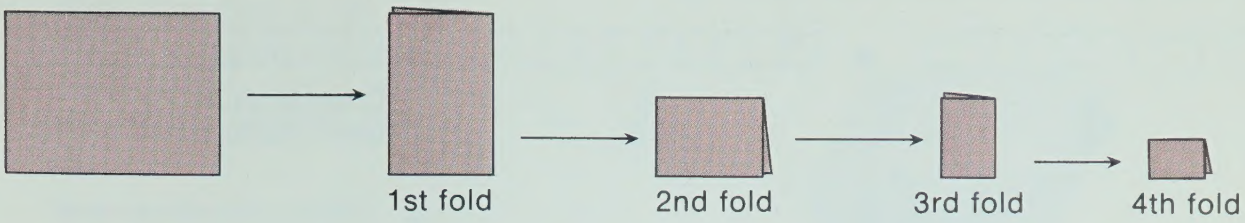
Answers will vary.

_____ sq units

Each "A" figure can be put on a geoboard and related to the previous geoboard page. This activity will assist children in making better estimates for area.

Fractions in Measurement

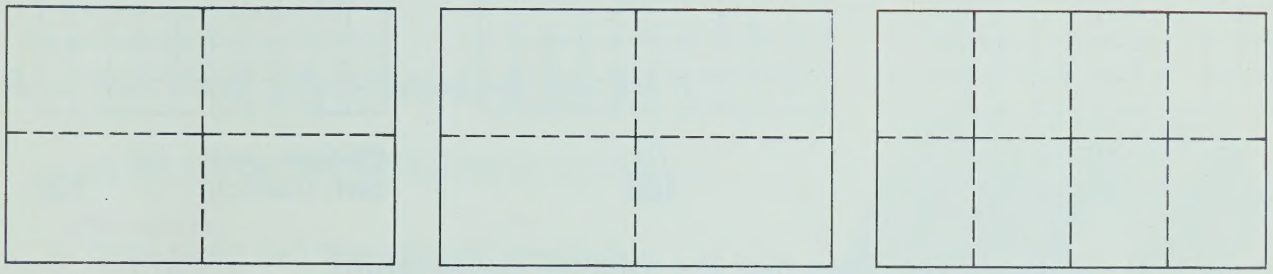
Fold a paper in half 4 times.



Guess how many parts you will see when you unfold the paper. _____

Now unfold the paper. How many parts do you see? 16

Color enough parts to show the fraction. Different parts may be colored.



1 of 4 parts

$\frac{1}{4}$

3 of 4 parts

$\frac{3}{4}$

3 of 8 parts

$\frac{3}{8}$

$\frac{1}{8}$

$\frac{1}{16}$

$\frac{8}{16}$

$\frac{15}{16}$

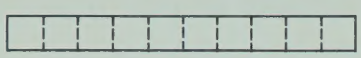
$\frac{\square}{\square}$ Answers will vary. $\frac{\square}{\square}$

You choose the fraction and color the parts.

Exploring Tenths and Hundredths

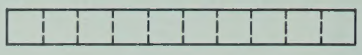
1. Color enough parts to show the fractions.

A



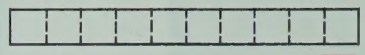
$\frac{1}{10}$

B



$\frac{5}{10}$

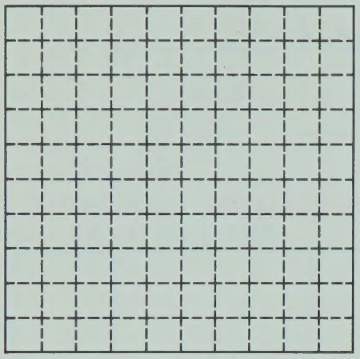
C



Choose your own fraction. $\frac{\square}{10}$

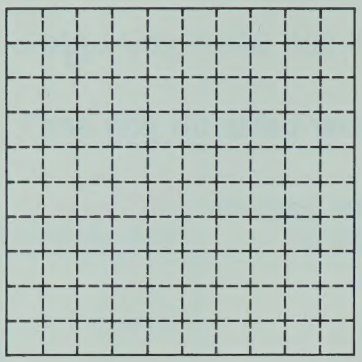
Answers will vary.

D



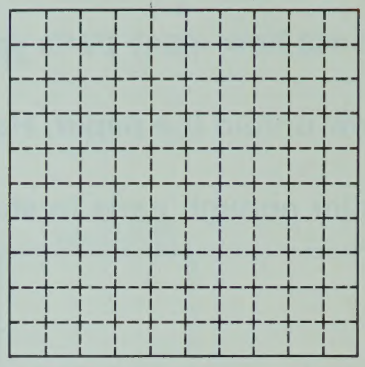
$\frac{10}{100}$

E



$\frac{75}{100}$

F



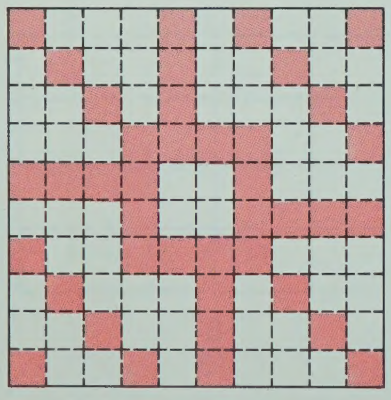
Choose your own fraction. $\frac{\square}{100}$

Answers will vary.

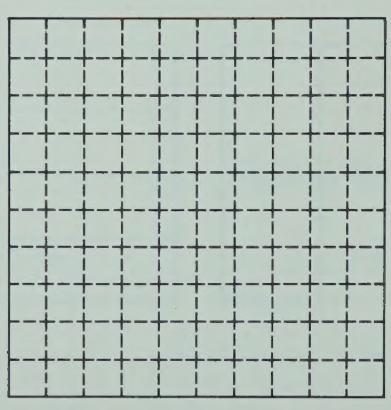
2. Color an interesting design. Then give the fraction (using tenths or hundredths) that shows what part is colored. Answers will vary.

EXAMPLE: $\frac{40}{100}$

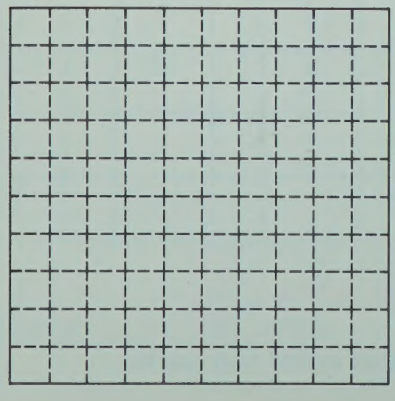
A



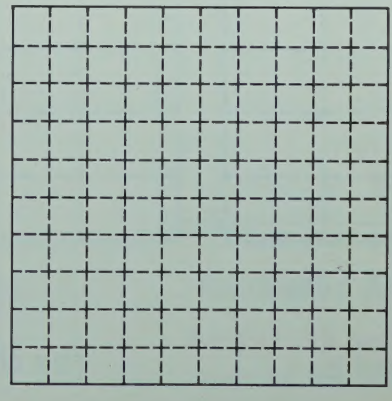
B



C



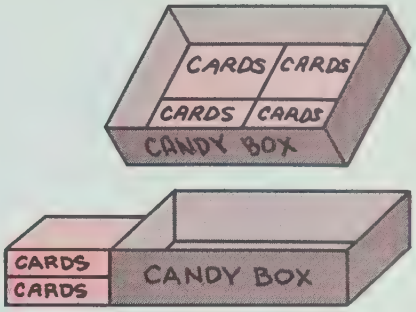
D



Give freedom and challenge to create interesting designs in exercise 2. Check only that children have the correct fraction for the part they colored.

Some Box Problems

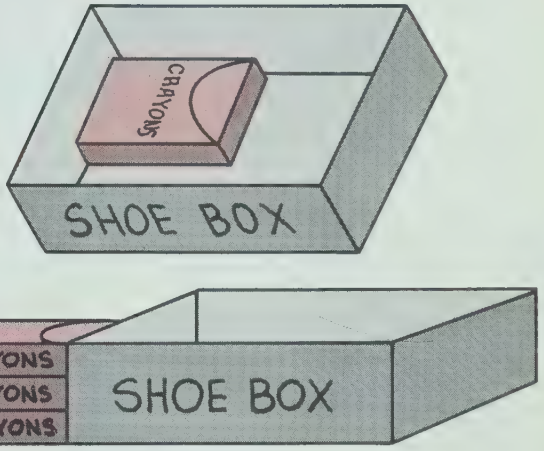
1. Four card boxes cover the bottom of this candy box. The card boxes can be stacked two deep. How many card boxes will the candy box hold? 8



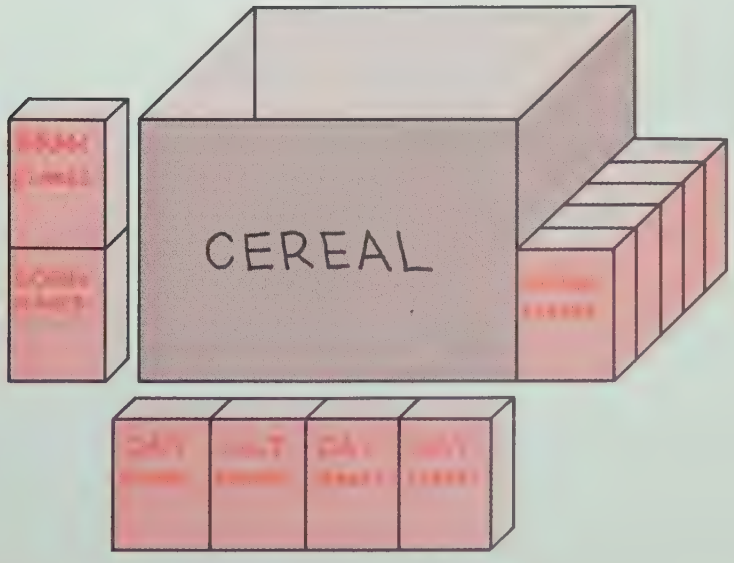
2. A How many crayon boxes are needed to fill the bottom of the shoe box? 4

B How deep can the crayon boxes be stacked? 3

C How many boxes of crayons can be put into the shoe box? 12



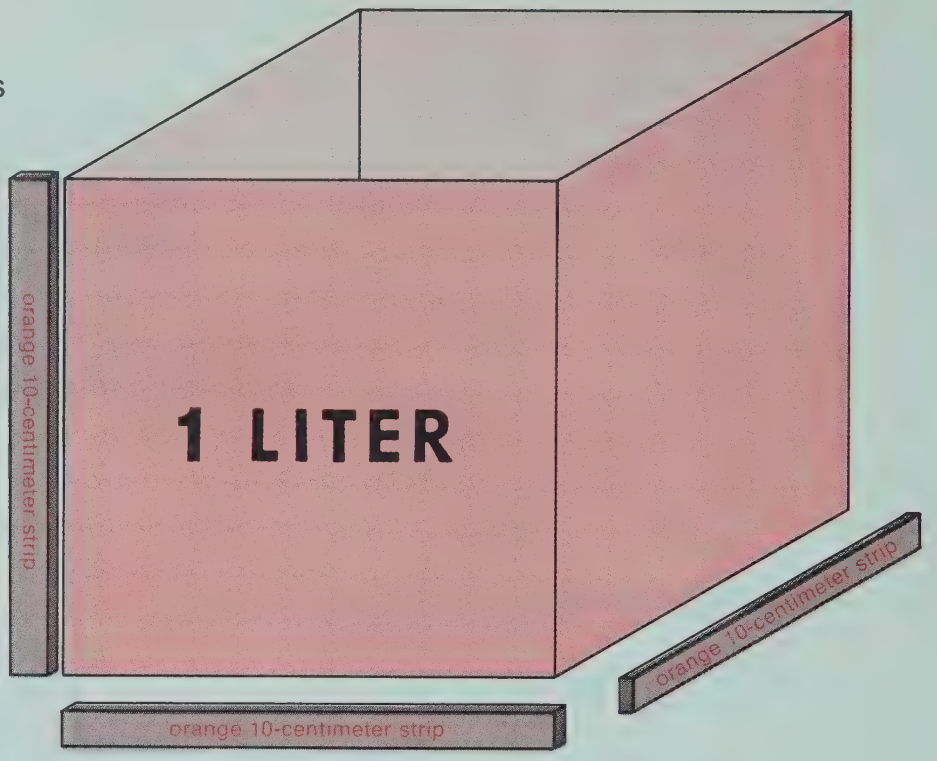
3. How many small cereal boxes will the large cereal box hold? 40



Actually placing boxes or cubes in a box will help children visualize the idea in these exercises.

How Much Will It Hold?

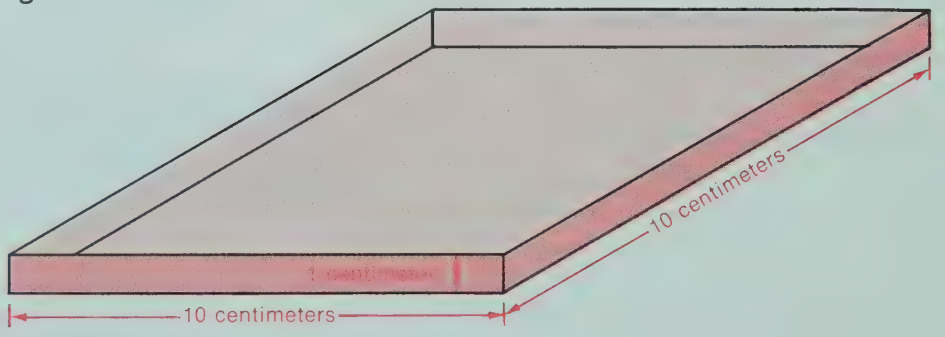
A cube-shaped container with each edge as long as the 10-centimeter strip holds 1 liter of liquid.



Complete the following:

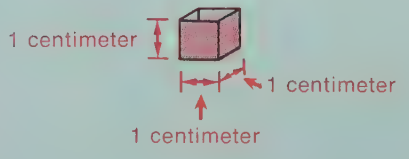
1. How many of this container full of water would it take to fill the liter?

10



2. How many of this container full of water would it take to fill the liter? 100

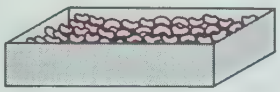
3. How many of this container full of water would it take to fill the liter? 1000



Building at least one layer (10 × 10 × 1) of the liter with centimeter cubes (White Cuisenaire rods) will help children gain a feel for the "size" of a liter. For children who can handle the concepts, pages 94, 95, and 96 may be used to reinforce and extend measurement in the metric system.

Beans, Beansticks, and Boxes

Can you find 3 ways to group the beans using beansticks and boxes. Use the **fewest** number of "containers" in one row. Mark a ✓ beside this row.

Amount		Boxes (100 beans) 	Beansticks (10 beans) 	Beans 0	
1. 236 beans (two hundred thirty-six)	A	2	3	6	✓
	B	other answers will vary			
	C				
2. 341 beans (three hundred forty-one)	A	3	4	1	✓
	B	other answers will vary			
	C				
3. 184 beans (one hundred eighty-four)	A	1	8	4	✓
	B	other answers will vary			
	C				
4. 752 beans (seven hundred fifty-two)	A	7	5	2	✓
	B	other answers will vary			
	C				
5. 913 beans (nine hundred thirteen)	A	9	1	3	✓
	B	other answers will vary			
	C				

Be sure children understand that the boxes represent 100 and the beansticks represent 10.

Estimating One Hundred

Estimate each of the following. Use the code at the bottom of the page to check your estimate. Then "grade" your estimate.

1. One hundred drops of water would

fill what part of a glass? $\frac{1}{4}$ depends on size of glass

My estimate was

very close _____ not too far off _____ far off _____

2. One hundred days is about

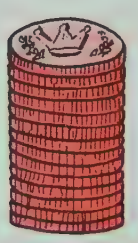
how many weeks? 14

My estimate was

very close _____ not too far off _____ far off _____

3. How many centimeters tall is

a stack of 100 checkers? 60



My estimate was

very close _____ not too far off _____ far off _____

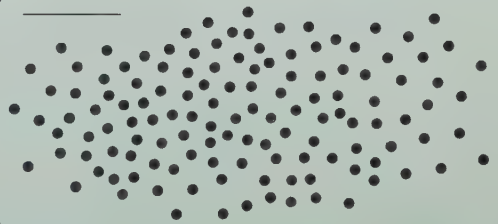
4. A stack of 100 pennies is as tall

as how many pennies stacked on edge? 8

My estimate was

very close _____ not too far off _____ far off _____

5. Circle your estimate of 100 dots.
Count to check your estimate.
My estimate was



very close _____ not too far off _____ far off _____

Answers will vary.

---

1. $\frac{P}{V}$ 2. PV 3. ZH 4. D

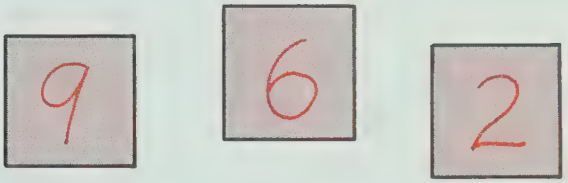
Code: P = 1 R = 2 T = 3 V = 4 X = 5 Z = 6 B = 7 D = 8 F = 9 H = 0

Three-Digit Numerals

1. Think of all the 3-digit numerals using these digits. List them in order from smallest to largest.

358 (smallest) 385 538 583 835 853 (largest)

2. Think of all the 1, 2, and 3 digit numerals using these three digits. List them in order from smallest to largest.



2 (smallest) 6 9 26 29
62 69 92 96 269
296 629 692 926 962 (largest)

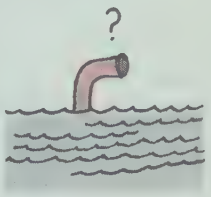
3. I am the smallest 3-digit number with all digits different and even. What number am I?

204

4. I am the largest 3-digit number with all digits different and odd. What number am I?

975

5. I am the smallest 3-digit number with all digits alike. What number am I?



111

For the first two exercises, it will be helpful if the children "make" all the numbers first, before trying to write them in order. You may need to review even and odd numbers for two of the riddles.

Estimating 1000

Can you circle the best estimate for each question?
Check your guess using the code at the bottom of this page.

1. 1000 hours is about ___?___ days.

A less than 10 B between 40 and 50 C more than 100

2. About how many of your classmates on the scales together would weigh 1000 kilograms?

A less than 15 B between 20 and 60 C more than 100

3. A 1000 page book (with pages like your regular math book) would be ___?___ centimeters thick.

A about 10 B more than 20 C about 5

4. How long is 1000 seconds?

A about 1 hour B about 15 minutes C 2 or 3 days

5. How high is a stack of 1000 pennies?

A less than 1 meter B between 1 and 2 meters C more than 2 meters

---

Code: X = 1 P = 2 J = 3 B = 4 R = 5 D = 6 Z = 7 N = 8 C = 9 K = 0

1. BP 42 2. About BK 40 3. B $\frac{X}{P}$ 4$\frac{1}{2}$ 4. XZ 17 5. XDK cm. 160

As a follow-up to this lesson you could have children actually demonstrate 1000 of some objects of their choosing.

Digit Riddles — 4-Digit Numbers

1. I am the smallest 4-digit number with all digits alike and even.

What number am I?

2 2 2 2

2. I am the smallest 4-digit number with all digits even and different. 0 is even and I have a zero.

What number am I?

2 0 4 6

3. I am the smallest 4-digit number with all digits odd and different.

What number am I?

1 3 5 7

4. I am the largest 4-digit number with all digits odd and alike.

What number am I?

9 9 9 9

5. I am the largest 4-digit number with all digits odd and different.

What number am I?

9 7 5 3

6. I am the largest 4-digit number with all digits different.

What number am I?

9 8 7 6

7. I am the smallest 4-digit number with all digits different.

What number am I?

1 0 2 3

8. I am a 4-digit number. All of my digits are different and their sum is 22.

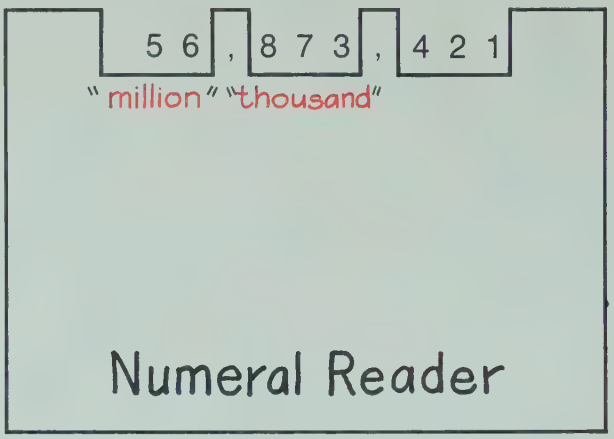
Answers will vary.

What number am I?

2 3 8 9

Reading and Writing Larger Numbers

Cut out this **numeral reader** and use it to read and write numerals in the exercises below.



Read "fifty-six million,
eight hundred seventy-three thousand,
four hundred twenty-one."

1. Read these numerals:

A In a recent year the population of New York City was
1 1 , 4 1 0 , 0 0 0. eleven million, four hundred ten thousand

B The average distance from earth to the sun is about
1 4 8 , 7 2 9 , 2 0 0 kilometers.
one hundred forty-eight million, seven hundred twenty-nine thousand, two hundred

2. Use your "numeral reader" to write these numerals.

A Average distance to the Moon: Three hundred ninety-two thousand, one hundred seventy-one kilometers. 392,171

B One of the largest checks every written:
Three hundred thirty-four million,
eight hundred sixty-seven thousand, eight hundred seven dollars. $334,867,807

C You choose a large number to write and read. Answers will vary.

Pairs or small groups for the reading activity involved in this lesson.

Creating Equations

Write 10, 20, 30, 40, 50, 60, 70, 80, 90, or 100 in each ☐. Write another one of these numbers in each ☐. Then solve the equation.

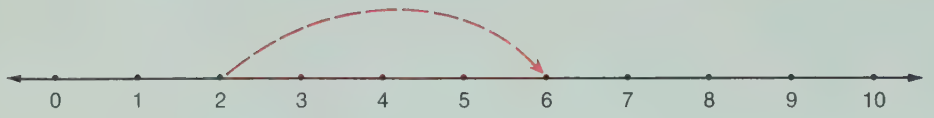
Write a subtraction equation that goes with each addition equation.

1. 40 + 60 = 100 — 100 − 60 = 40
2. ☐ + ☐ = ☐ — answers will vary
3. ☐ + ☐ = ☐ — answers will vary
4. ☐ + ☐ = ☐ — answers will vary
5. ☐ + ☐ = ☐ — answers will vary
6. ☐ + ☐ = ☐ — answers will vary
7. ☐ + ☐ = ☐ — answers will vary
8. ☐ + ☐ = ☐ — answers will vary

Help children to see that for each addition equation they write there are two corresponding subtraction equations. In other words a child could write 100 − 40 = 60 for number 1 and still be correct.

Operations on the Number Line

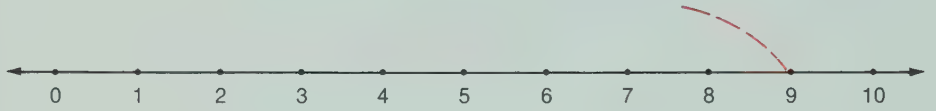
Draw a jump on the number line. Then complete the equation. Answers will vary.

1. 

2 + ☐ = ☐

2. 

40 + ☐ = ☐

3. 

9 − ☐ = ☐

4. 

80 − ☐ = ☐

5. 

900 − ☐ = ☐

6. 

3000 + ☐ = ☐

7. 

6000 − ☐ = ☐

8. 

120 − ☐ = ☐

9. 

70 + ☐ = ☐

Since children are writing their own equations be sure that the equation they write coincides with the figure they have drawn.

Breaking a Code

Write the correct number in each ☐. Find this number on the "Message Sheet" and write the letter of the problem in the blank above it.

A $9 + 6 =$ 15

B $3 + 7 = 10$

C $8 + 6 = 14$

D $12 - 7 =$ 5

E $15 - 6 =$ 9

F $17 - 4 =$ 13

G $19 - 5 =$ 14

H $12 - 4 = 8$

I $15 - 7 = 8$

J $19 - 9 =$ 10

K $13 - 2 = 11$

L $17 - 8 = 9$

M $16 - 6 = 10$

N $11 - 3 = 8$

O $12 - 5 = 7$

P $17 - 9 = 8$

Q $30 + 40 =$ 70

R $80 - 60 =$ 20

S $150 - 70 =$ 80

T $130 - 40 =$ 90

U $140 - 80 =$ 60

V $23 - 5 =$ 18

W $29 - 10 =$ 19

X $50 - 20 =$ 30

Y $100 - 50 = 50$

Z $26 - 5 =$ 21

MESSAGE SHEET

S	O	M	E	O	N	E		I	S		A
80	12	16	9	12	11	9		7	80		15

R	E	A	L	L	Y		F	I	N	E
20	9	15	8	8	50		13	7	11	9

P	R	O	B	L	E	M		S	O	L	V	E	R	!
17	20	12	3	8	9	16		80	12	8	18	5	20	

As a follow-up to this lesson encourage children to make code and message of their own.

Exploring Basic Principles

1. Fill in each gray square ▢.

2. Find 2 "same number" squares and color them the same color.

3. Do this until the table is complete. Use as many colors as you can.

4. What principle does this show?

order principle

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	3 + 6	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	6 + 3	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

5. Find the sums.

A $10 + (5 + 4) =$ 19	**B** $20 + (3 + 2) =$ 25	**C** $40 + (5 + 5) =$ 50
$(10 + 5) + 4 =$ 19	$(20 + 3) + 2 =$ 25	$(40 + 5) + 5 =$ 50
$15 + 4 =$ 19	$23 + 2 =$ 25	$45 + 5 =$ 50
D $10 + (5 + 6) =$ 21	**E** $20 + (8 + 7) =$ 35	**F** $50 + (8 + 6) =$ 64
$(10 + 5) + 6 =$ 21	$(20 + 8) + 7 =$ 35	$(50 + 8) + 6 =$ 64
$15 + 6 =$ 21	$28 + 7 =$ 35	$58 + 6 =$ 64

6. What basic principle could be used to make the work easy in problem **5**? grouping principle

You may wish to expand the ideas on this page by providing additional experiences with the order and grouping principles.

Rearranging Addends

1. Find the sums.

A

2	7 → 9	
4	3 → 7	
6	10 → 16	

B

3	6 → 9	
7	4 → 11	
10	10 → 20	

C

4	5 → 9	
6	3 → 9	
10	8 → 18	

D

9	8 → 17	
1	2 → 3	
10	10 → 20	

E

20	60 → 80	
80	10 → 90	
100	70 → 170	

F

300	400 → 700	
700	600 → 1300	
1000	1000 → 2000	

2. Find the sums. Look for (10) 's.

A 7, 6, 3, 4 — (10), (10) — 20

B 9, 5, 1, 5 — 10, 10 — 20

C 8, 6, 2, 3 — 10 — 19

D 4, 6, 5, 3 — 10 — 18

E 8, 2, 6, 4, 3 — 10, 10 — 23

F 7, 4, 3, 6, 8 — 10, 10 — 28

G 9, 10, 4, 1, 5, 6 — 10 — 25

H 3, 2, 10, 8, 5, 10, 2 — 20

Encourage children to figure out by themselves how the tables at the top of the page actually work. The basic idea for the exercises on this page is that the addends can be rearranged and the sum will remain the same.

Mathematics Magic

1. Find the sum of the numbers in each row, in each column, and along each diagonal of the squares below.

A

8	1	6	15
3	5	7	15
4	9	2	15
15	15	15	15

B

5	2	11	18
12	6	0	18
1	10	7	18
18	18	18	18

C

50	20	50	120
40	40	40	120
30	60	30	120
120	120	120	120

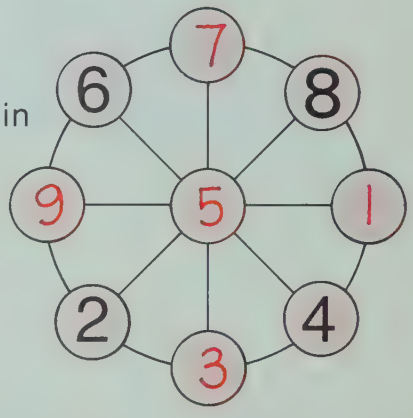
Why do you think these are called MAGIC SQUARES?

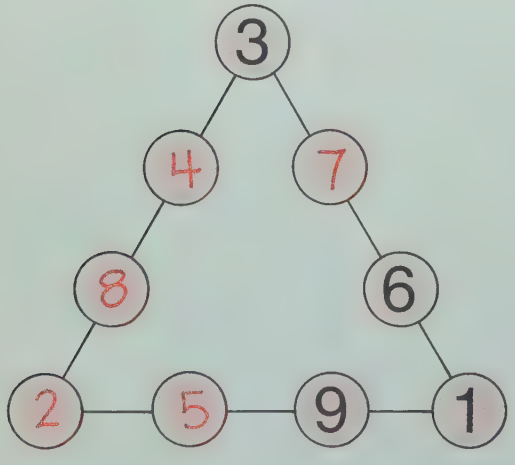
4	3	8
9	5	1
2	7	6

2. Complete to make a magic square with "magic sum" 15.

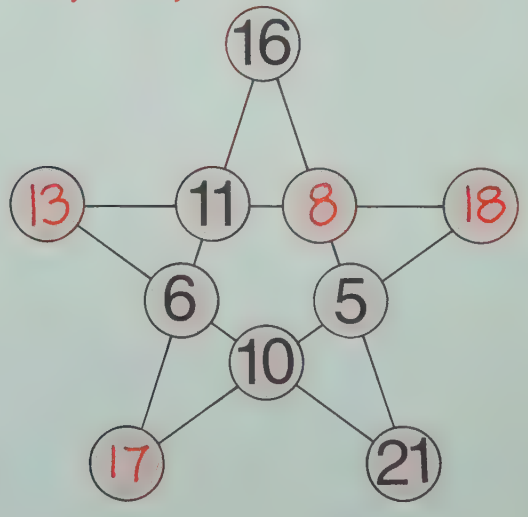
3. Make a "magic circle." The sum of 3 numbers in each line across the circle must be 15. Use each of the numbers 1 through 9.

arrangement of 7 and 3 and 9 and 1 may vary





4. Make a "magic triangle." The sum of the 4 numbers on each side must equal 17. Use each of the numbers 1 through 9.



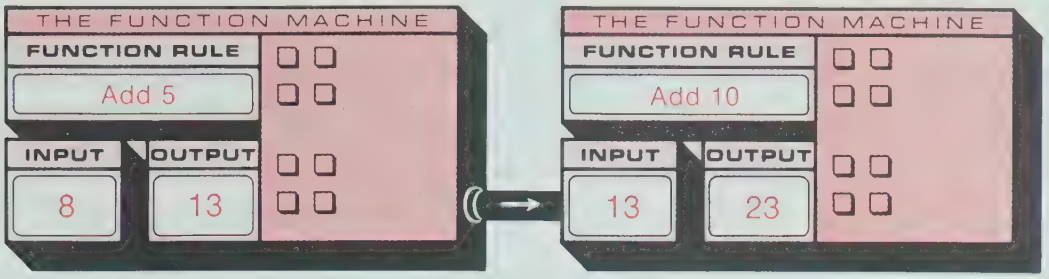
5. Make a "magic star." Each line of 4 numbers must add to 50.

You may choose to have the children answer the question under the magic squares orally rather than writing it out.

Function Machine Fun

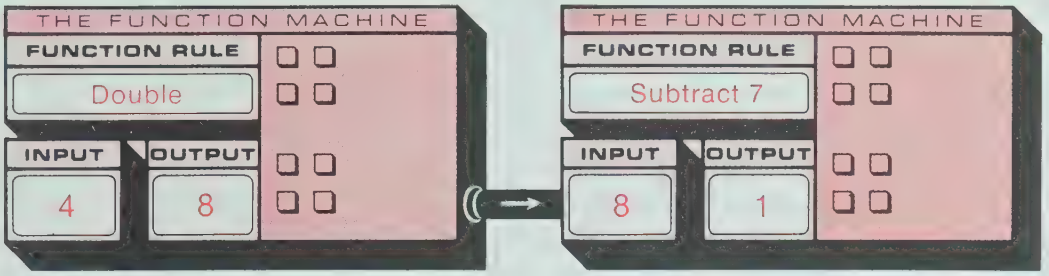
Two function machines are connected so that the output of the first machine automatically becomes the input of the 2nd machine. Give the numbers in the table or the missing rule for each exercise.

1.



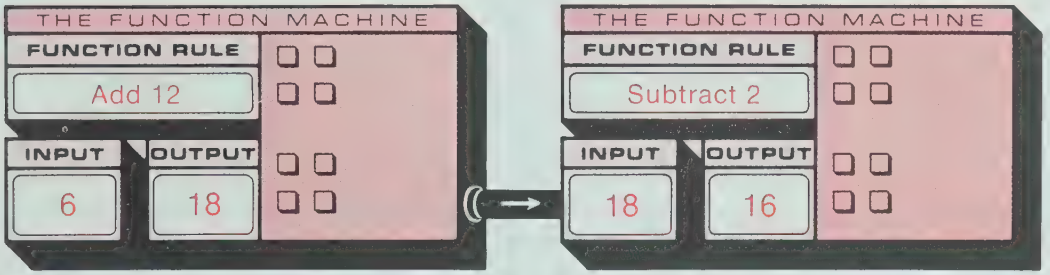
	First Input	Final Output
	8	23
A	5	20
B	50	65

2.



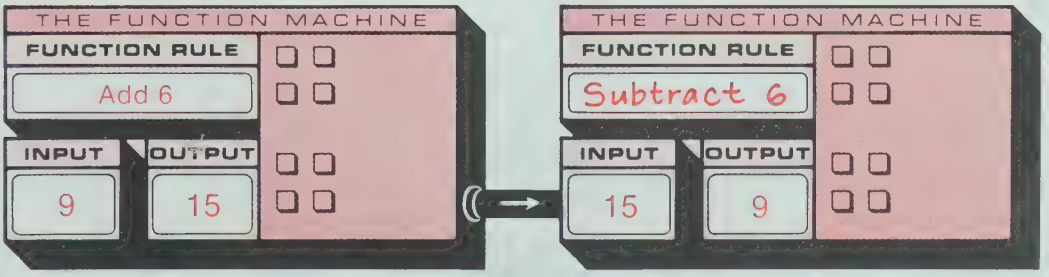
	First Input	Final Output
	4	1
A	10	13
B	50	93

3.



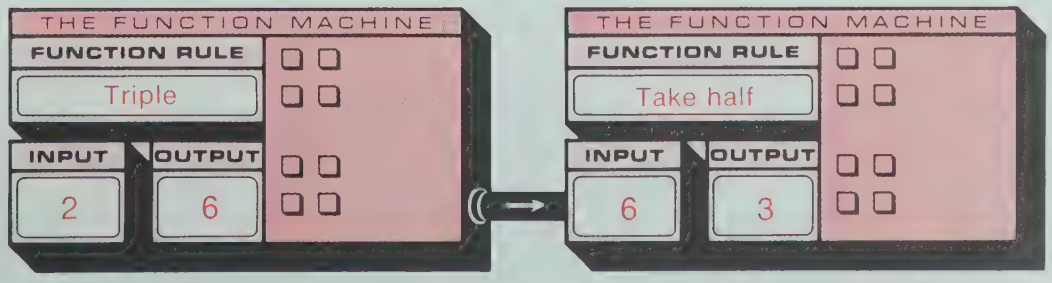
	First Input	Final Output
	6	16
A	9	19
B	5	15

4.



	First Input	Final Output
	9	9
	5	5
	3	3

5.

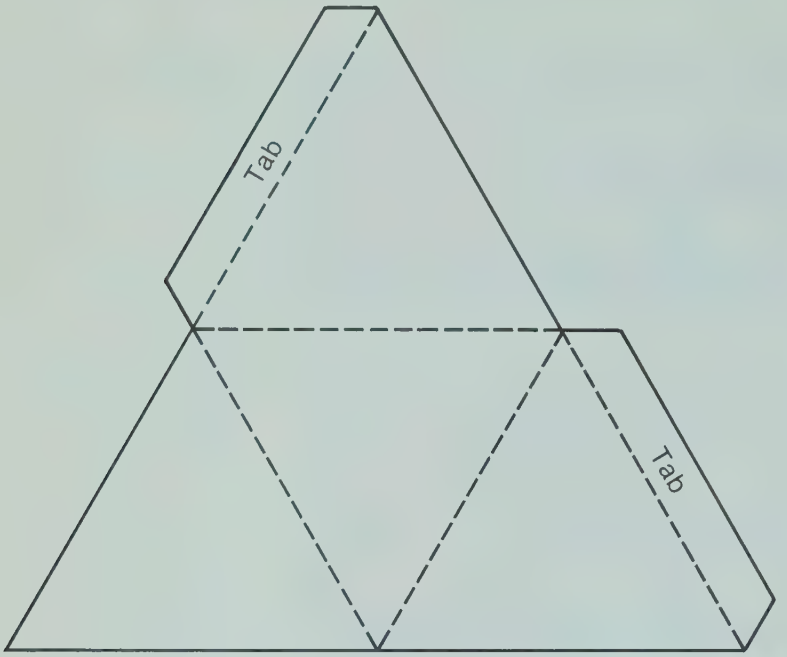


	First Input	Final Output
	2	3
A	4	6
B	6	9

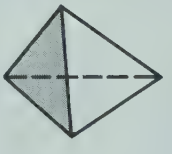
Encourage children to extend this activity by inventing function rules of their own and giving them to classmates.

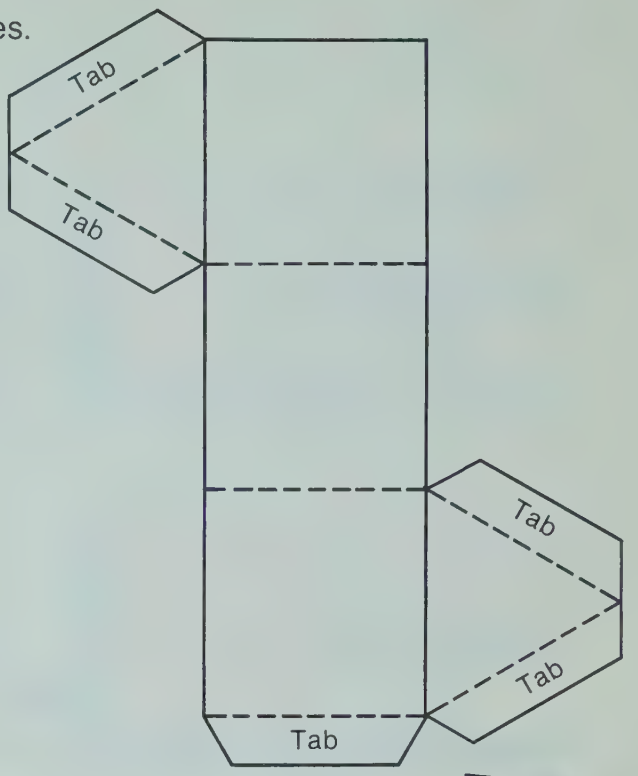
Exploring Space Figures

1. Cut out the patterns and make the space figures. Use them to complete the table.

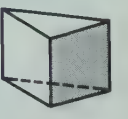


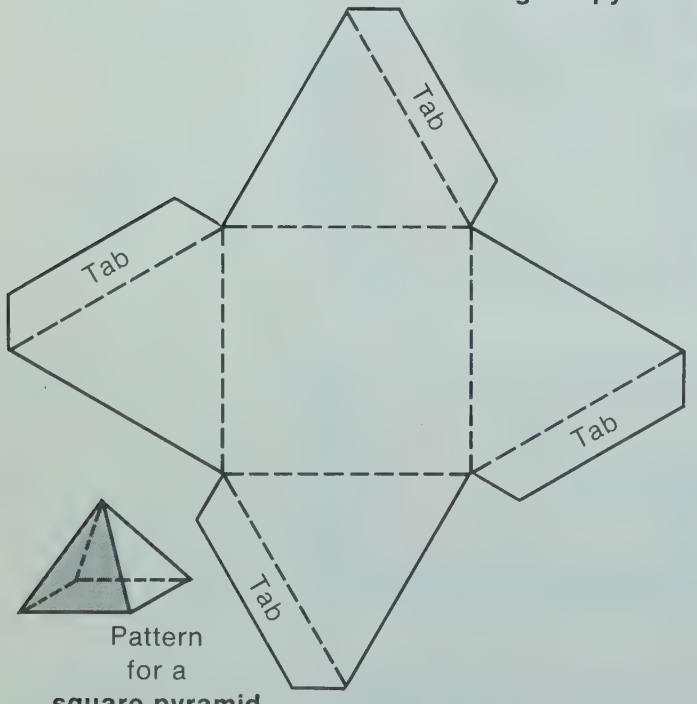
Pattern for a **triangular pyramid**





Pattern for a **prism**





Pattern for a **square pyramid**

Space Figure	Number of Faces	Number of Vertices	Number of Edges
Cube	6	8	12
Triangular Pyramid	4	4	6
Prism	5	6	9
Square Pyramid	5	5	8

2. Add the number of **faces** and the number of **vertices** for a figure.

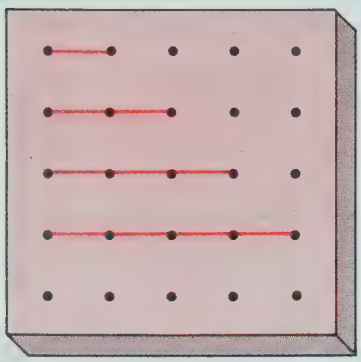
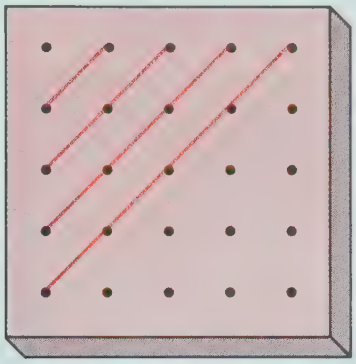
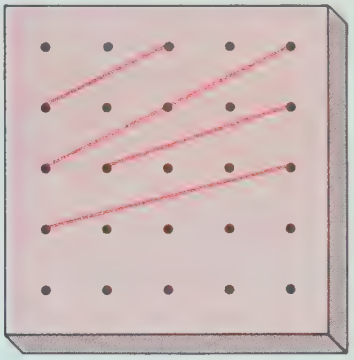
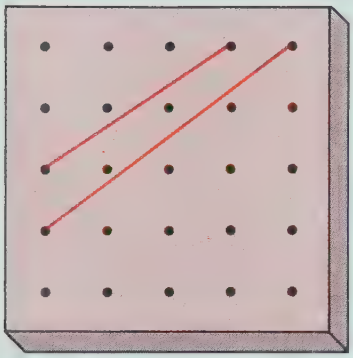
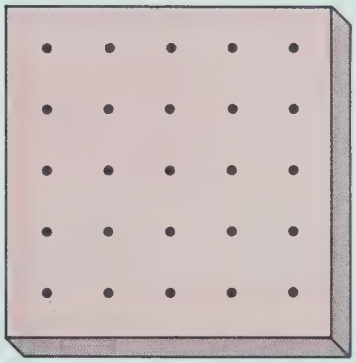
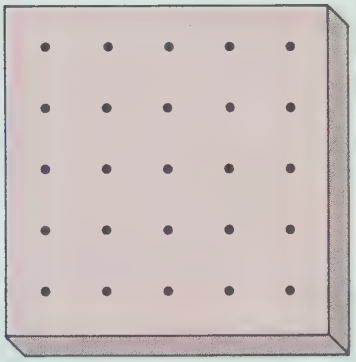
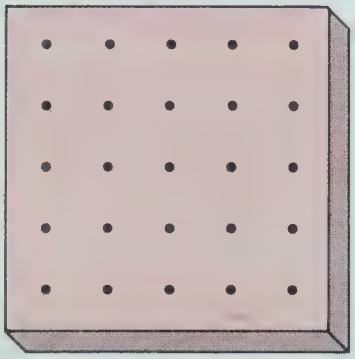
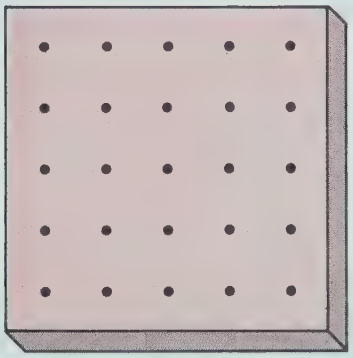
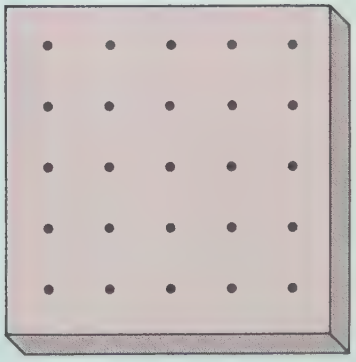
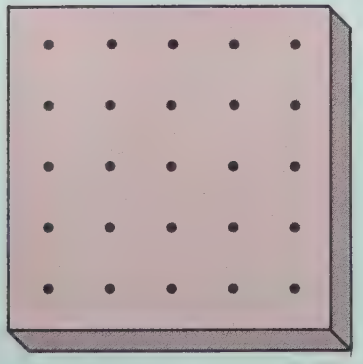
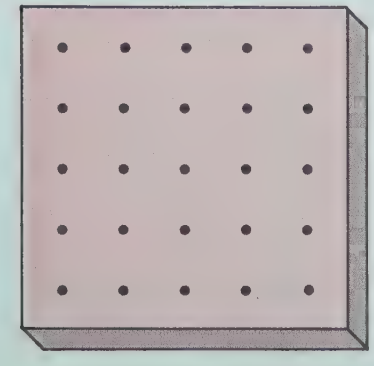
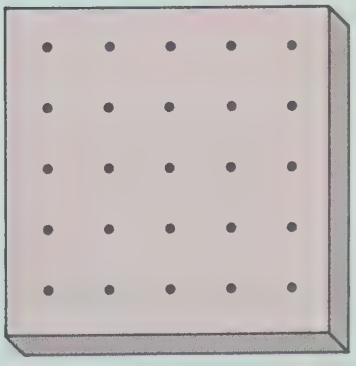
How close is the sum to the number of **edges**? Two more than the number of edges.

Is this true for each figure? Yes

Children should cut very accurately around all figures. Instruct them to fold on the dotted lines and use paste or tape to attach the tabs.

Segments on the Geoboard

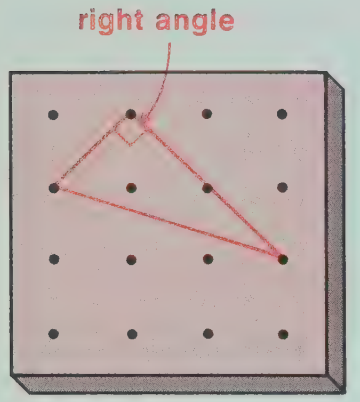
Show a different length segment on each "geoboard." Use tracing paper to compare the lengths. Then put a "1" beside the longest segment, a "2" beside the next longest, and so on.

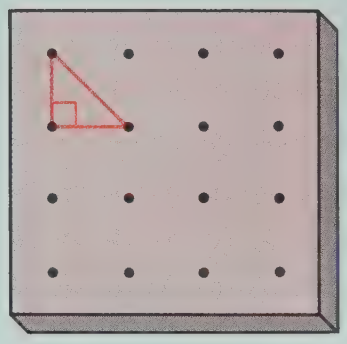













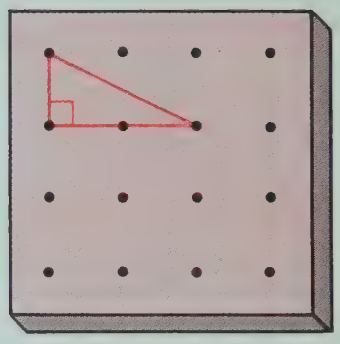
There are 14 different length segments that can be drawn on the 5 × 5 geoboard. It is not intended that children be able to find all 14 segments.

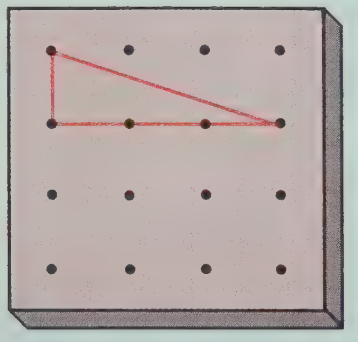
Comparing Triangles

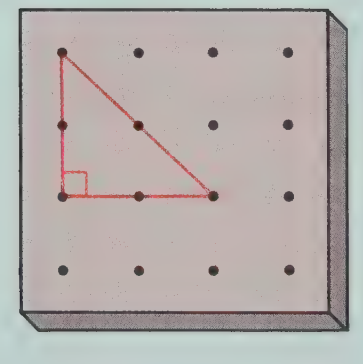
Draw a triangle that has a **right angle** on each geoboard. Each triangle must be different in size or shape. How many can you draw?

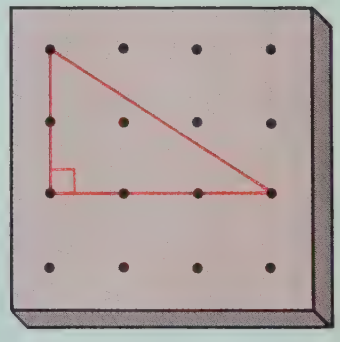


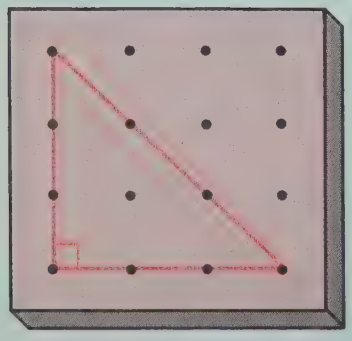


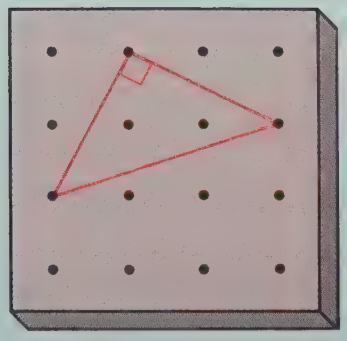


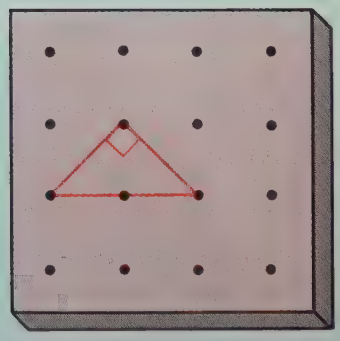








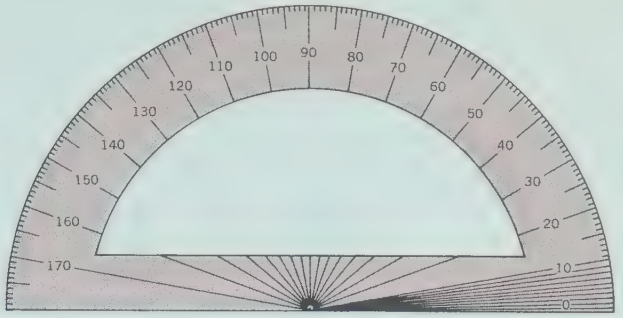




There are 9 different triangles that can be drawn that are different in size or shape. Don't expect children to find them all.

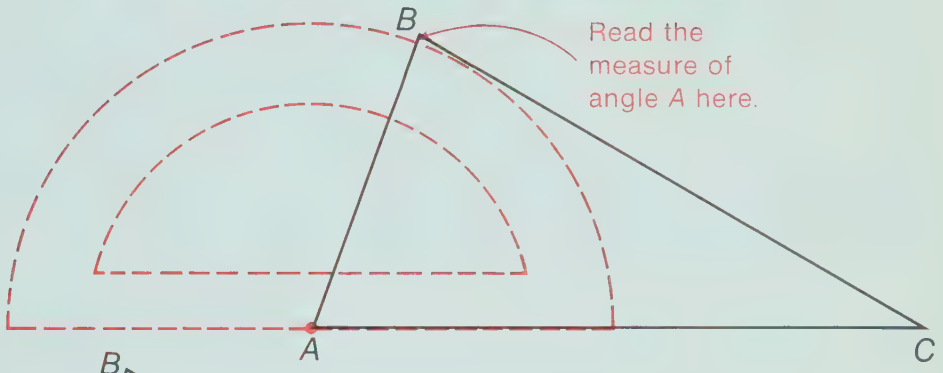
Measuring Angles

A **protractor** is used to measure angles. Cut out this **protractor** and place it in the dotted outlines to measure the angles of triangle *ABC*



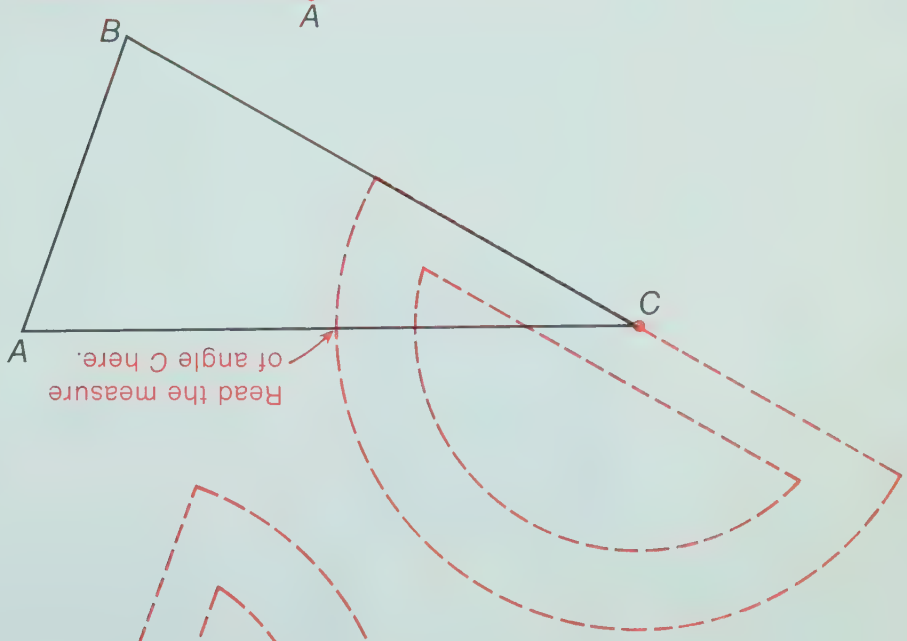
1. The measure of angle *A* is

about 70 degrees.



2. The measure of angle *C*

is about 30 degrees.



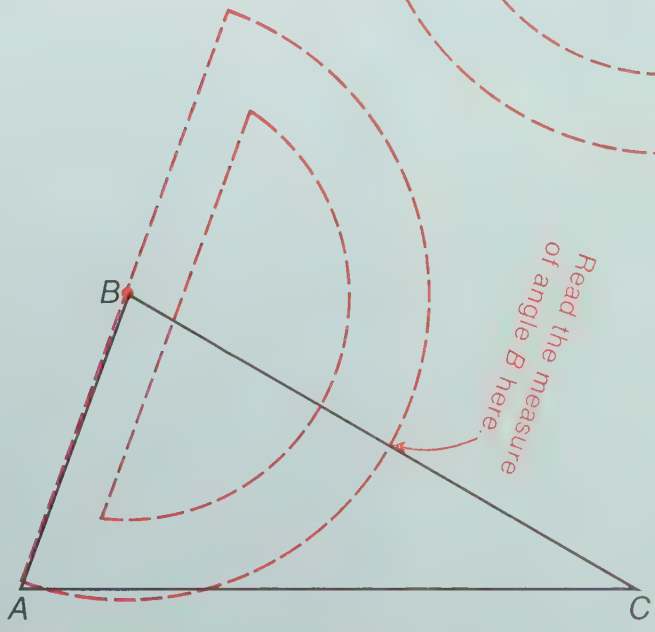
3. The measure of angle *B*

is about 80 degrees.

4. Is the sum of the 3 angle measures close to 180 degrees?

Yes

5. Draw another triangle of your own and find the measures of its angles.

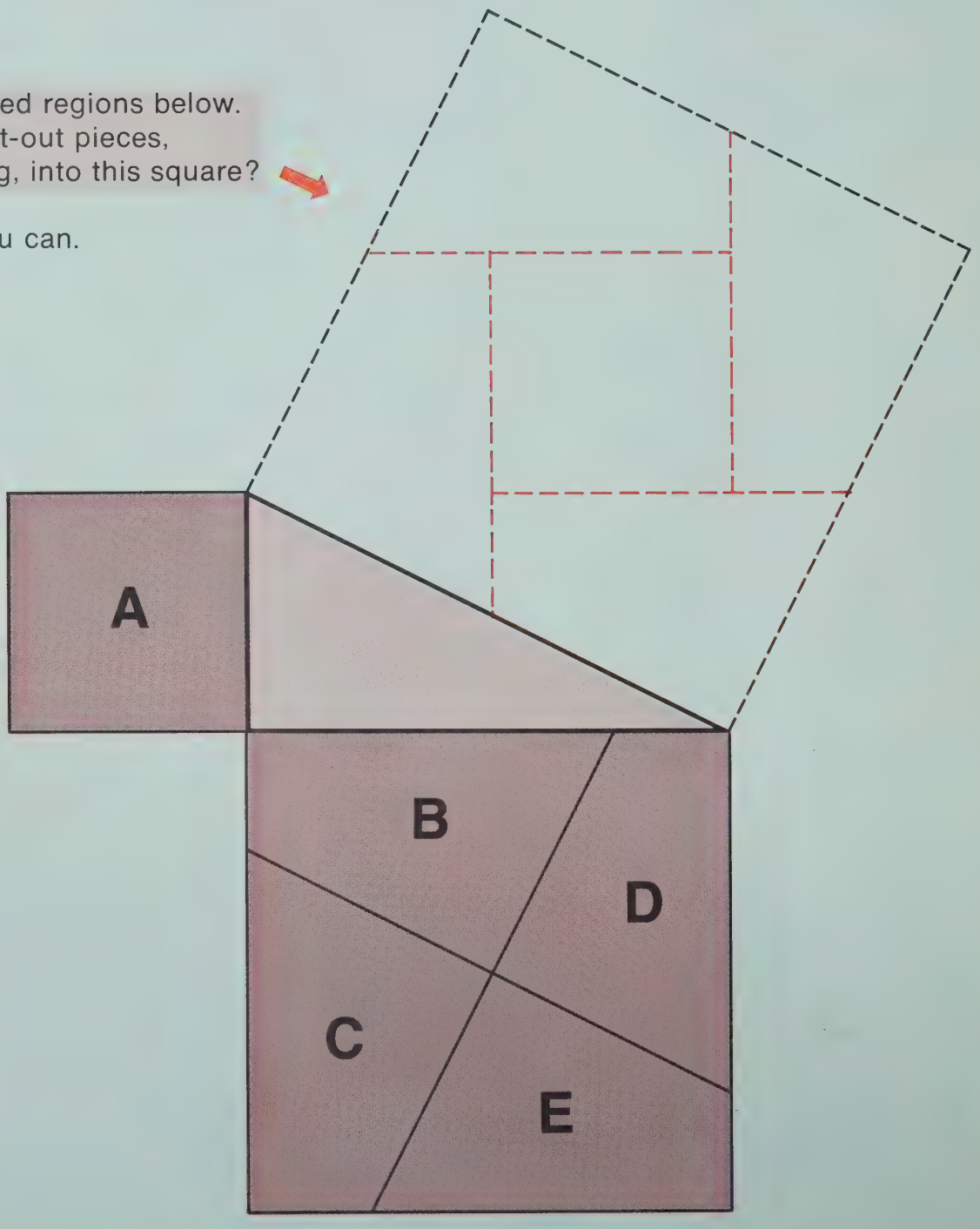


Make sure that children place the protractor as accurately as possible when measuring the angles. If it is done carefully the sum of the angles of triangle *ABC* should be close to 180 degrees.

A Right Triangle Puzzle

Cut out the 5 lettered regions below.
Can you fit all 5 cut-out pieces,
with no overlapping, into this square?

Paste them in if you can.



This right triangle puzzle should prove challenging for most children. This lesson is an informal introduction to the concept of the Pythagorean theorem: The sum of the squares of the two legs of a right triangle is equal to the square of the hypotenuse.

Money Problems

Can you find 3 ways to count out each amount of money. For one of the ways use the **fewest** number of "money pieces." Mark a ✓ beside the row that shows this way.

Amount		Ten Dollar Bills (1000 pennies) 	Dollars (100 pennies) 	Dime (10 pennies) 	Pennies 
1.	**A**		2	20	36
436¢	**B**		4	3	6
	C				
2.	**A**		other answers will vary		
2563¢	**B**	2	5	6	3
	C				
3.	**A**				
9429¢	**B**	9	4	2	9
	C		other answers will vary		
4.	**A**				
1294¢	**B**	1	2	9	4
	C				
5.	**A**		other answers will vary		
5708¢	**B**	5	7	0	8
	C		other answers will vary		

Adding and Subtracting

1. Find the sums. Look for (100) 's.

A

60
90 → 100
40
70 → 100
10
+30 → 100
300

B

80
60 ⟩ 100
40
20 ⟩ 100
70
+10
280

C

30
90 ⟩ 100
60
70 ⟩ 100
80
+40
370

D

50
30 ⟩ 100
30
50 ⟩ 100
40
+70
270

E

90
80
70 ⟩ 100
10
20 ⟩ 100
+90
360

2. Write the numeral in each ▢.

A

364
+5▢3
887

B

436
+921
1357

C

501
+▢01
1002

D

985
−206
779

E

▢67
−3▢5
542

F

359
−1▢8
251

G

▢76
−276
200

H

999
+2▢1
1230

3. Find the sums and missing numbers.

A

600	50	7	→ 657
200	50	2	→ 252
▢00	100	9	→ 909

B

400	30	9	→ 439
500	40	7	→ 547
900	70	16	→ 986

Fun with Sums

The words MOM, BOB, and RADAR remain the same when the letters are written in reverse order. They are called **word palindromes**. Numbers such as 1551, 464, and 81618 remain the same when the digits are written in reverse order. They are called **number palindromes**. Study the example. Try some. Keep going until you arrive at a palindrome.

EXAMPLE:

```
  39   ← Start with any number.
+ 93   ← Reverse the digits.
-----
 132   ← Add.
+231   ← Reverse the digits.
-----
 363   ← The sum is a palindrome!
```

```
  94
+ 49
-----
 143
+341
-----
 484
```

```
  83
+ 38
-----
 121
```

```
  59
+ 95
-----
 154
+451
-----
 605
+506
-----
1111
```

```
  79
+ 97
-----
 176
+671
-----
 847
+748
-----
1595
+5951
-----
7546
+6457
-----
14003
+30041
-----
44044
```

```
  174
+ 471
-----
 645
+ 546
-----
1191
+1911
-----
3102
+2013
-----
5115
```

```
  78
+ 87
-----
 165
+561
-----
 726
+627
-----
1353
+3531
-----
4884
```

```
  182
+ 281
-----
 463
+ 364
-----
 827
+ 728
-----
1555
+5551
-----
7106
+6017
-----
13123
+32131
-----
45254
```

Try some of your own on another sheet of paper.

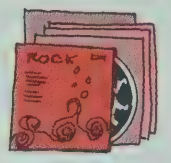
Creating Story Problems

Write a story problem for each picture that can be solved using addition or subtraction. Then give your problems to a classmate to solve.

Sample answers:

How much further is Urban Center than Peakville ?

278 - 135 = 143



Records
$3.98

Book
$4.35

How much would 1 record and 1 book cost?

$3.98 + $4.35 = $8.33

Monkey
62 Kilograms

Bear
377 Kilograms

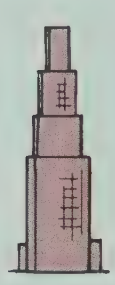
Tiger
183 Kilograms

Deer 125 Kilograms
Lion 157 Kilograms

How much do the Lion and the Monkey weigh?

157 + 62 = 219

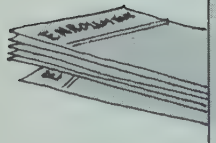
Empire State Building
381 meters



Sears Building
442 meters

John Hancock Center
344 meters

How much taller is the Empire State Building than the John Hancock Center? 381-344=37



JEFFERSON SCHOOL ENROLLMENT	
Grade 1	123
Grade 2	96
Grade 3	176
Grade 4	198
Grade 5	84
Grade 6	117

How much larger is grade 3 enrollment than grade 2 enrollment? 176 - 96 = 80

Height in centimeters	
Jane	108
Bill	120
Tom	135
Kathy	116
Sue	126
Jan	138
Fred	142

How much taller is Jan than Bill?

138 - 120 = 18

Fun with Subtraction

1. Study the example. Complete three more.

You choose a number.

EXAMPLE:

8 3 5	← Choose a number with 3 different digits.
−5 3 8	← Reverse the digits.
2 9 7	← Subtract.
+7 9 2	← Reverse the Digits.
1 0 8 9	← Add.

4 5 2	
254	
198	
891	
1089	

9 5 3	
359	
594	
495	
1089	

What did you discover? The result is always 1089.

2. Study the example. Complete three more.

You choose a number.

EXAMPLE:

6 2 4	← Choose a number with 3 different digits.
−4 2 6	← Reverse the digits.
1 9 8	← Subtract the smaller number from the larger.
	← Notice the middle digit.
9	← Add the two outside digits.

9 5 3	
359	
594	
9	

7 6 3	
367	
396	
9	

What did you discover? The middle digit and the sum of two outside digits is always 9.

3. Study the examples. Do some of your own. Then check by subtracting the usual way.

EXAMPLES:

5 2 → Add 3 → 5 5	8 3 → Add 4 → 8 7	5 3 → Add 6 → 5 9
−2 7 → Add 3 → −3 0	−4 6 → Add 4 → −5 0	−2 4 → Add 6 → −3 0
2 5 ← 2 5	3 7 ← 3 7	2 9 ← 2 9

4 3 →	7 2 →	9 1 →
−2 8 → ___	−5 6 → ___	−7 5 → ___
←	← answers will vary	←

Note that when the children perform the subtraction in each exercise they may need to rearrange the numbers in order to subtract the smaller from the larger.

Making and Checking Estimates

Complete at least one of the projects below.

1. How much does your class weigh?

CALCULATION SPACE

A Your estimate: answers will vary.

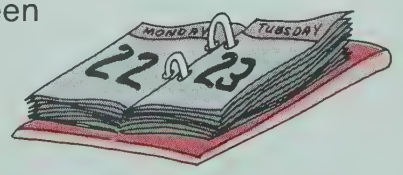
B Calculated Answer: answers will vary

2. Who weighs most—all the persons in your class with last names starting with the letters A-M—or those with N-Z? How much more?

A Your estimate: answers will vary

B Calculated Answer: answer will vary.

3. How many days have you been alive?



A Your estimate: answers will vary

B Calculated Answer: answers will vary

4. How tall, in centimeters, are five of your friends?

A Your estimate: answers will vary.

B Calculated answer: answers will vary.

Choosing and Ordering Purchases

Suppose you have $200 to spend. Use a mail order catalog and choose what you would buy. Fill out this order form and calculate the total amount. Answers will vary.

ORDER FORM

Name _____
(First Name) (Middle Initial) (Last Name)

Address _____

City _____ State _____ Zip _ _ _ _ _

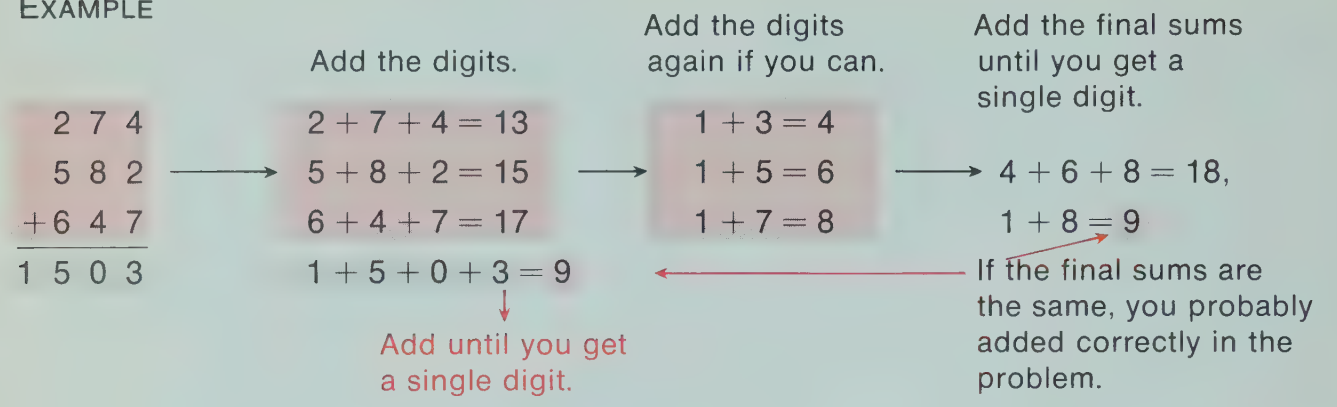
Telephone Number _____ Area Code _____

Page Number	Name of Item	How sold? (Each, package, etc.)	How Many?	Catalog Number	Total Price
				Total Amount	

Adding and Subtracting Quick-Checks

1. Study the example. Then find the sums and use the method shown to check your answers.

EXAMPLE



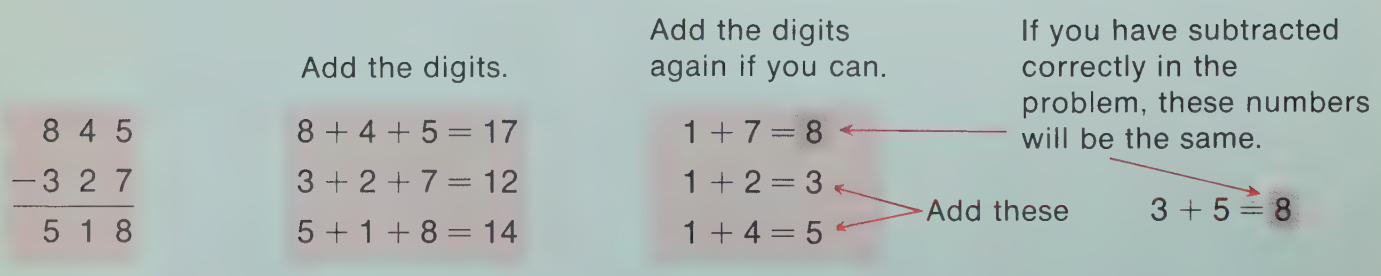
A $\begin{array}{r} 473 \\ 268 \\ +947 \\ \hline 1688 \end{array}$ → 14 → 5, 16 → 7, 20 → 2, 14 → 5, 23 → 5

B $\begin{array}{r} 874 \\ 1369 \\ +\ 498 \\ \hline 2741 \end{array}$ → 19 → 10 → 1, 19 → 10 → 1, 21 → 3, 14 → 5

C $\begin{array}{r} 647 \\ 984 \\ +392 \\ \hline 2023 \end{array}$ → 17 → 8, 21 → 3, 16 → 7, 14 → 5, 7

2. Study the example. Then find the differences and use the method shown to check your answers.

EXAMPLE



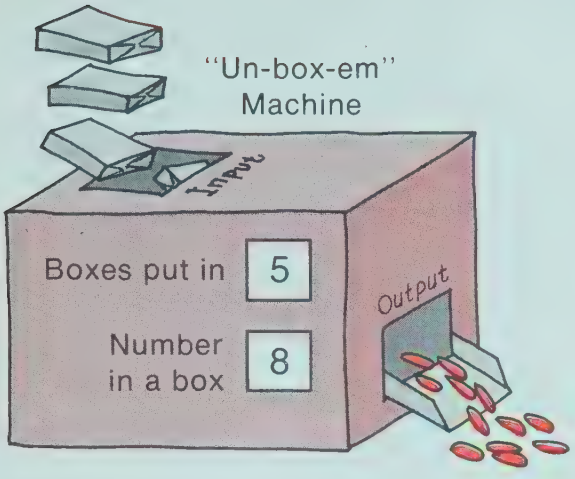
A $\begin{array}{r} 934 \\ -267 \\ \hline 667 \end{array}$

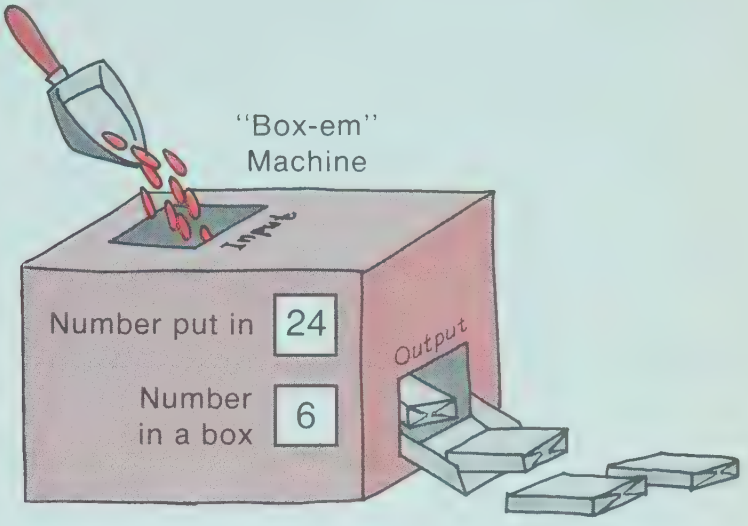
B $\begin{array}{r} 784 \\ -359 \\ \hline 425 \end{array}$

C $\begin{array}{r} 1093 \\ -\ 768 \\ \hline 325 \end{array}$

Note that if the final sums are the same you probably added correctly but it is not certain. What is certain is that if the final sums are different you did not add correctly. In other words it is possible to have an incorrect answer and yet have the digital sums be the same.

Box Problems





When a box of things is put into the "Un-box-em" Machine, it takes them out of the box. When you put things in the "Box-em Machine, it places them in boxes. Complete the following tables.

1.

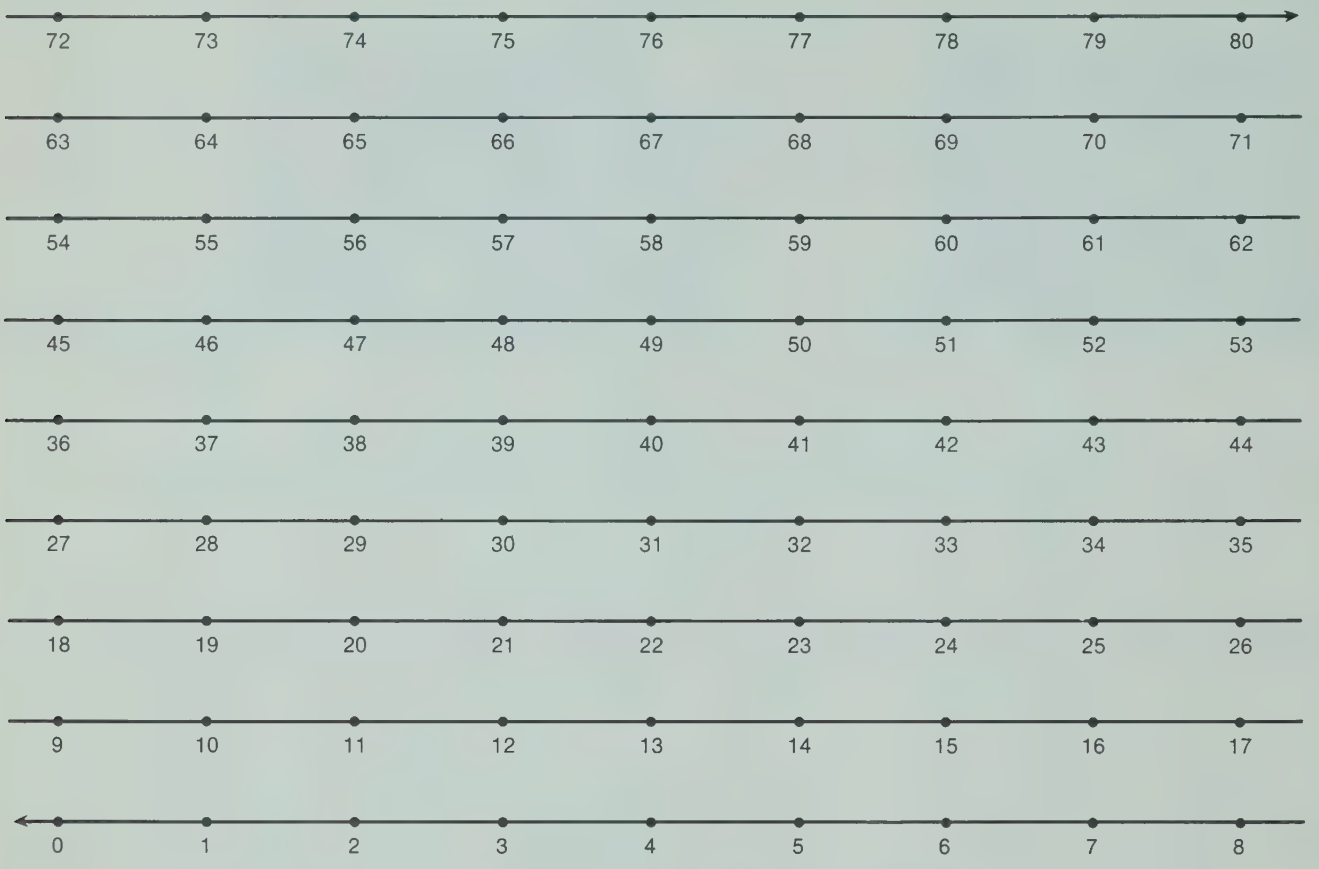
	Record for the "Un-box-em" Machine			
	Item	Input (number of boxes)	Number in each box	Output (number of items)
A	Books	6	4	24
B	Marbles	8	9	72
C	Candy Bars	10	6	60
D	Crayons	4	8	32
E	Eggs	6	12	72
F	Mini-Cars	9	4	36
G	Coins	7	6	42
H	Cookies	10	12	120
I	Stamps	5	10	50
J	Erasers	8	5	40

2.

	Record for the "Box-em" Machine			
	Item	Input (number of items)	Number in each box	Output (number of boxes)
A	Pencils	24	6	4
B	Apples	20	4	5
C	Gum	35	5	7
D	Lollipops	32	8	4
E	Cards	18	6	3
F	Baseballs	36	12	3
G	Candy Bars	48	6	8
H	Books	28	4	7
I	Marbles	60	10	6
J	Watches	12	4	3

Special Number Line Jumps

This number line has been stacked in sections so it will fit on the page. Study the examples to see how to make jumps on the number line. Then complete the tables.



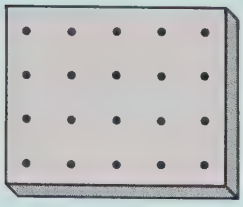
Symbol	0 ↑2	12 ←3	19 ↓1	18 →5	0 →5	0 ↑6	72 ↓1	72 ↓8	8 ←7	0 ↑3	0 ↑4	80 ↓8
Start	0	12	19	18	0	0	72	72	8	0	0	80
Land	18	9	10	23	5	54	63	0	1	27	36	8
					A	B	C	D	E	F	G	H

Symbol	0 ↗1	28 ↘2	4 ↖3	20 ↘2	0 ↗5	0 ↗8	8 ↖3	8 ↖8	1 ↗4	5 ↗2	25 ↖1	18 ↘2
Start	0	28	4	20	0	0	8	8	1	5	25	18
Land	10	12	28	0	50	80	32	72	41	25	35	2
					A	B	C	D	E	F	G	H

Avoid insofar as possible explaining the meaning of the symbols. Give the children every opportunity to discover on their own how these symbols relate to the jumps. Enough information is given in the two tables for them to make these discoveries.

How Many Nails?

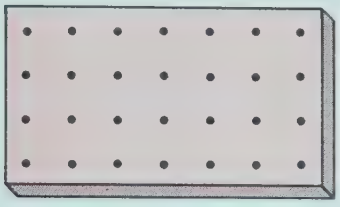
Tell how many nails on each board (including those covered) and write a multiplication equation for each.

1. 

Number of nails 20

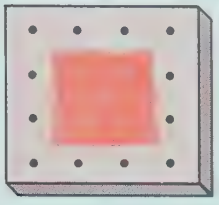
Equation: 4 × 5 = 20

number of rows; number of nails in each row

2. 

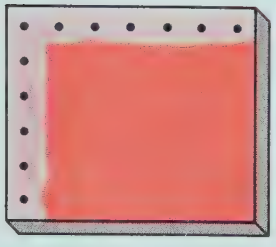
Nails: 28

Equation: 4 x 7 = 28

3. 

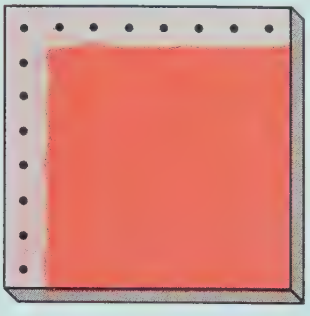
Nails: 16

Equation: 4 x 4 = 16

4. 

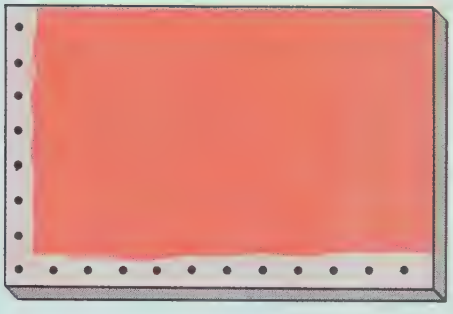
Nails: 42

Equation: 6 x 7 = 42

5. 

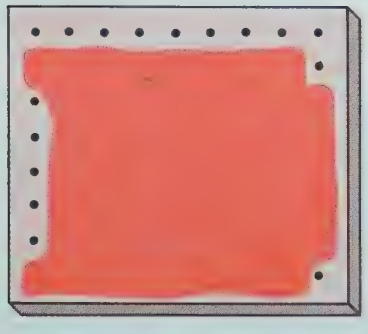
Nails: 64

Equation: 8 x 8 = 64

6. 

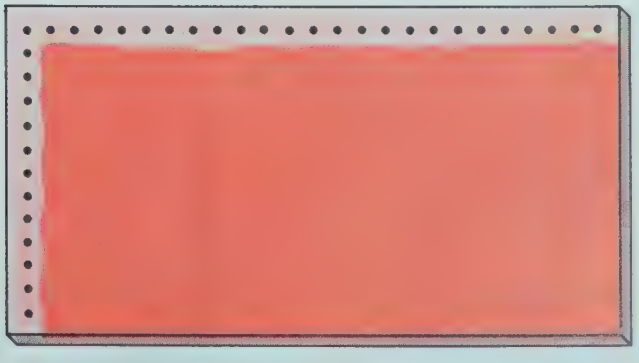
Nails: 96

Equation: 8 x 12 = 96

7. 

Nails: 72

Equation: 8 x 9 = 72

8. 

Nails: 325

Equation: 13 x 25 = 325

Make sure children understand there are the same number of nails in each row and the same number in each column. Children may need to use repeated addition in order to find the answer to the various multiplication equations.

Money Combinations

How many different amounts of money can you make by choosing one coin (or bill) from each set? Write the different amounts.

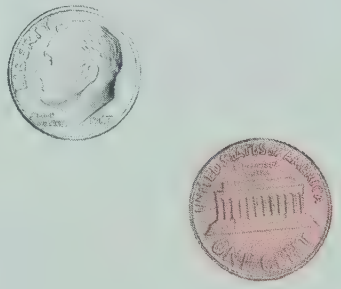
Complete the equations.

1. 1 copper coin × 4 "silver" coins = 4 different amounts

List the amounts here.

6¢
11¢
26¢
51¢

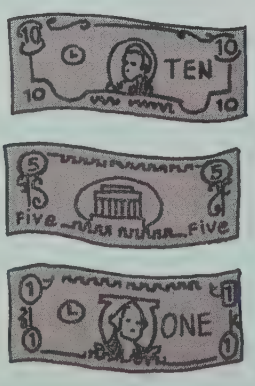
2. 3 large coins × 2 small coins = 6 different amounts



List the amounts here.

51¢
26¢
6¢
15¢
35¢
60¢

3. 3 bills × 5 coins = 15 different amounts



List the amounts here.

$10.50
$5.25
10.01
10.05
10.10
10.25
5.01
5.05
5.10
5.50
1.01
1.05
1.10
1.25
1.50

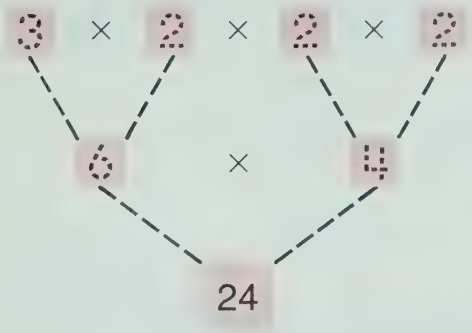
The number of different amounts children can make is given by the multiplication equation above each group of coins. You may wish to expand the lesson by asking them "How many amounts could you make if you had 4 bills and 5 coins" etc.

Factor Trees

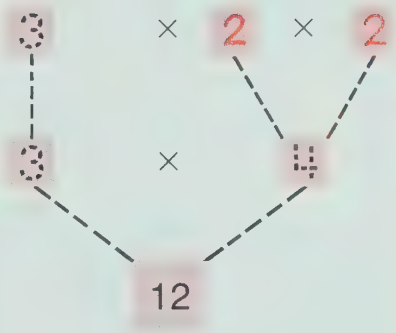
Help the "Factor Trees" grow as much as you can.

Rule: Do not use 1 in a factor tree.

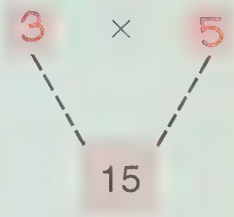
1.



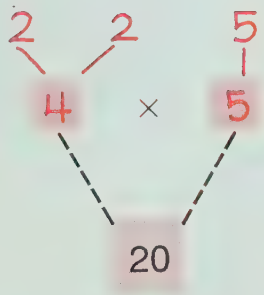
2.



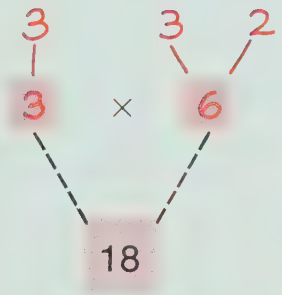
3.



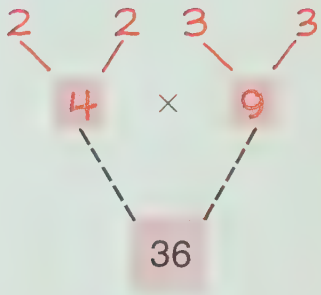
4.



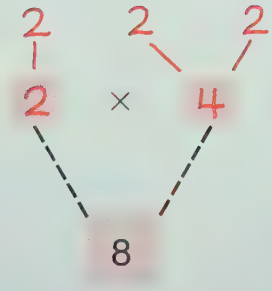
5.



6.

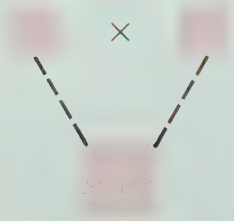


7.



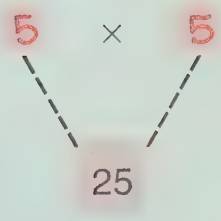
8.

Answers will vary



Make one of your own

9.

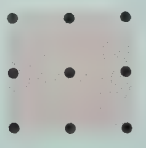


This may be the children's first introduction to Factor Trees. Have them study the example throughly before proceeding to the exercises. Note that in some exercises the numbers on the second row may be different but the top row will always contain the same numbers.

Multiplication Table Patterns

How many products can you find? Write them in the table.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	3 × 3 = 9	12	3 × 5 = 15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	5 × 3 = 15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81



1. Products like 3 × 3 are called **square numbers**.
Color all the square number boxes blue.

2. Find a row ↔ and a column ↕ in the table in which every box contains the same number. Color the row and the column yellow.

3. Color the row and the column that contain each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 brown.

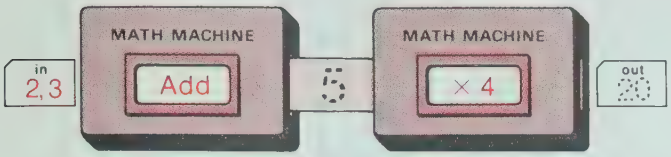
4. If the table were folded on the dotted line, then pairs of numbers such as 15 and 15 would match. Find as many pairs of matching numbers as you can and color the pairs with the same color. Use as many different colors for the pairs as you can.

By completing and coloring the multiplication table the children may recognize certain basic principles of numbers: Exercise 2—the multiplicative properties of zero; Exercise 3—the one principle; Exercise 4—the order principle.

A Math Machine

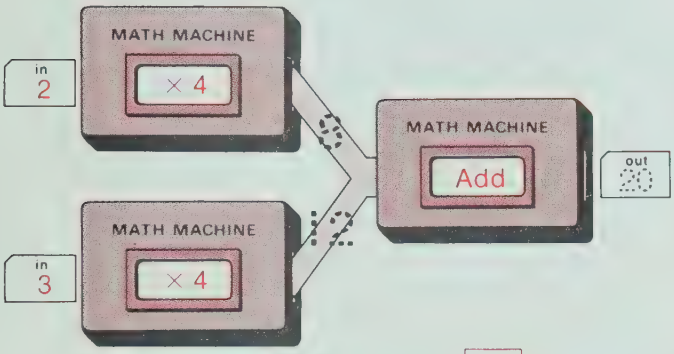
Give the output for each "Math Machine" hookup. Then complete the equation.

1. A



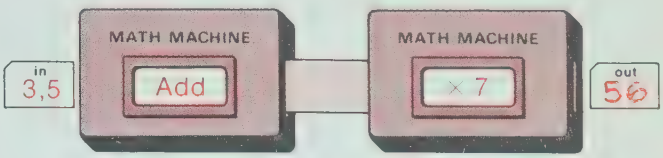
$(2 + 3) \times 4 = 20$

B



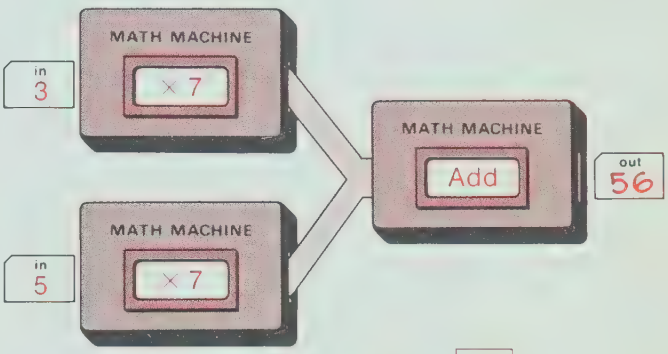
$(2 \times 4) + (3 \times 4) = 20$

2. A



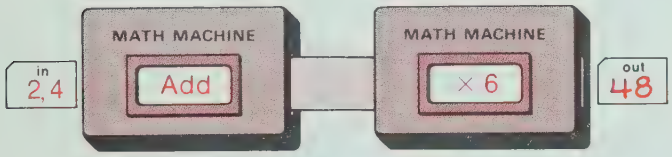
$(3 + 5) \times 7 = 56$

B



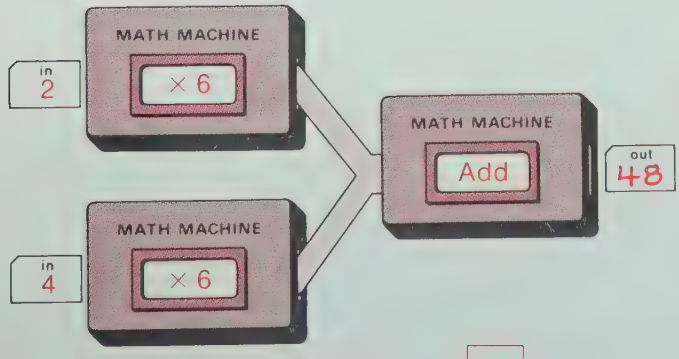
$(3 \times 7) + (5 \times 7) = 56$

3. A



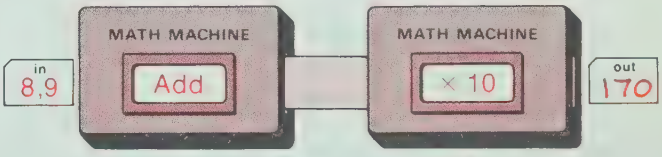
$(2 + 4) \times 6 = 48$

B



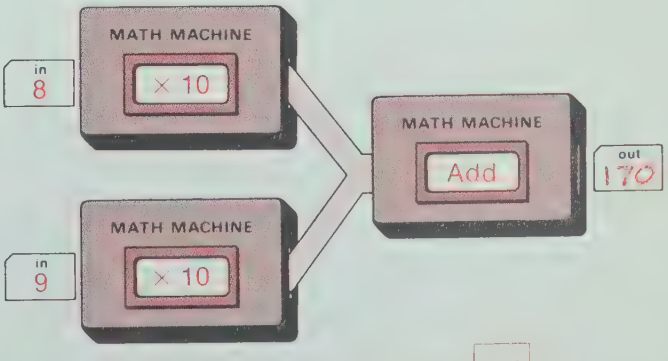
$(2 \times 6) + (4 \times 6) = 48$

4. A



$(8 + 9) \times 10 = 170$

B

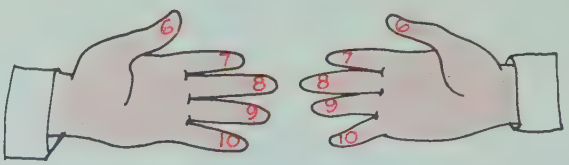


$(8 \times 10) + (9 \times 10) = 170$

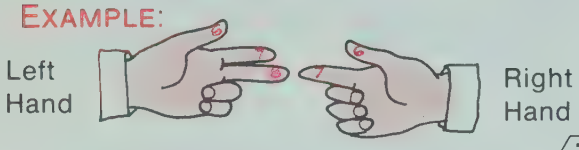
The Math Machine is a way of demonstrating the multiplication-addition principle.

Finger Multiplication

To make a finger calculator, label each finger as shown in the picture.
Study the example and then fill in the blanks.



1. EXAMPLE:



A How many open fingers (or thumbs)? ⬡5

B How many closed fingers on left? 2

C How many closed fingers on right? 3

D Product of closed fingers.

$8 \times 7 =$ ⬡5 □6

2.

A How many open fingers (or thumbs)? ⬡5

B How many closed fingers on left? 1

C How many closed fingers on right? 4

D Product of closed fingers.

$9 \times 6 =$ ⬡5 □4

3.

A How many open fingers (or thumbs)? ⬡3

B How many closed fingers on left? 3

C How many closed fingers on right? 4

D Product of closed fingers.

$7 \times 6 =$ ⬡3 □12 4

30 + 12 = 42

4.

A How many open fingers (or thumbs)? ⬡6

B How many closed fingers on left? 1

C How many closed fingers on right? 3

D Product of closed fingers.

$9 \times 7 =$ ⬡6 □3

5.

A How many open fingers (or thumbs)? ⬡4

B How many closed fingers on left? 2

C How many closed fingers on right? 4

D Product of closed fingers.

$8 \times 6 =$ ⬡4 □8

6.

A How many open fingers (or thumbs)? ⬡8

B How many closed fingers on left? 1

C How many closed fingers on right? 1

D Product of closed fingers.

$9 \times 9 =$ ⬡8 □1

7.

A How many open fingers (or thumbs)? ⬡7

B How many closed fingers on left? 0

C How many closed fingers on right? 3

D Product of closed fingers.

$10 \times 7 =$ ⬡7 □0

8. Try some on your fingers.

A How many open fingers (or thumbs)? ⬡

B How many closed fingers on left? _____

C How many closed fingers on right? _____

D Product of closed fingers.

_____ × _____ = ⬡ □

Note that in exercise 3 that the product of the closed fingers is 12. 12 should be thought of as one "ten and two". Thus the ten is moved to the tens place.

Facts About Facts

Complete each part.

$2 \times 9 = 18 \rightarrow 1 + 8 = \boxed{9}$

$3 \times 9 = 27 \rightarrow 2 + 7 = \boxed{9}$

$4 \times 9 = 36 \rightarrow 3 + 6 = \boxed{9}$

$5 \times 9 = 45 \rightarrow 4 + 5 = \boxed{9}$

$6 \times 9 = 54 \rightarrow 5 + 4 = \boxed{9}$

$7 \times 9 = 63 \rightarrow 6 + 3 = \boxed{9}$

$8 \times 9 = 72 \rightarrow 7 + 2 = \boxed{9}$

What did you discover? The sum of the digits is always 9.

$1 \times 8 = 8 \rightarrow \quad = \boxed{8}$

$2 \times 8 = 16 \rightarrow 1 + 6 = \boxed{7}$

$3 \times 8 = 24 \rightarrow 2 + 4 = \boxed{6}$

$4 \times 8 = 32 \rightarrow 3 + 2 = \boxed{5}$

$5 \times 8 = 40 \rightarrow 4 + 0 = \boxed{4}$

$6 \times 8 = 48 \rightarrow 4+8=12,\ 1 + 2 = \boxed{3}$ *

$7 \times 8 = 56 \rightarrow 5+6=11,\ 1 + 1 = 10 \rightarrow \boxed{2}$ *

$8 \times 8 = 64 \rightarrow 1 + 0 = \boxed{1}$ *

*Add the digits until you get a single digit to write in the box.

What did you discover? The sum decreases by 1 each time.

$2 \times 6 = 12 \rightarrow 1 + 2 = \boxed{3}$

$3 \times 6 = 18 \rightarrow 1 + 8 = \boxed{9}$

$4 \times 6 = 24 \rightarrow 2 + 4 = \boxed{6}$

$5 \times 6 = 30 \rightarrow 3 + 0 = \boxed{3}$

$6 \times 6 = 36 \rightarrow 3 + 6 = \boxed{9}$

$7 \times 6 = 42 \rightarrow 4 + 2 = \boxed{6}$

$8 \times 6 = 48 \rightarrow 4+8=12,\ 1 + 2 = \boxed{3}$

$9 \times 6 = 54 \rightarrow 5 + 4 = \boxed{9}$

$10 \times 6 = 60 \rightarrow 6 + 0 = \boxed{6}$

What did you discover? The numerals 3,9,6, are repeated in a pattern.

$1 \times 5 = 5 \rightarrow \quad = \boxed{5}$

$2 \times 5 = 10 \rightarrow 1 + 0 = \boxed{1}$

$3 \times 5 = 15 \rightarrow 1 + 5 = \boxed{6}$

$4 \times 5 = 20 \rightarrow 2 + 0 = \boxed{2}$

$5 \times 5 = 25 \rightarrow 2 + 5 = \boxed{7}$

$6 \times 5 = 30 \rightarrow 3 + 0 = \boxed{3}$

$7 \times 5 = 35 \rightarrow 3 + 5 = \boxed{8}$

$8 \times 5 = 40 \rightarrow 4 + 0 = \boxed{4}$

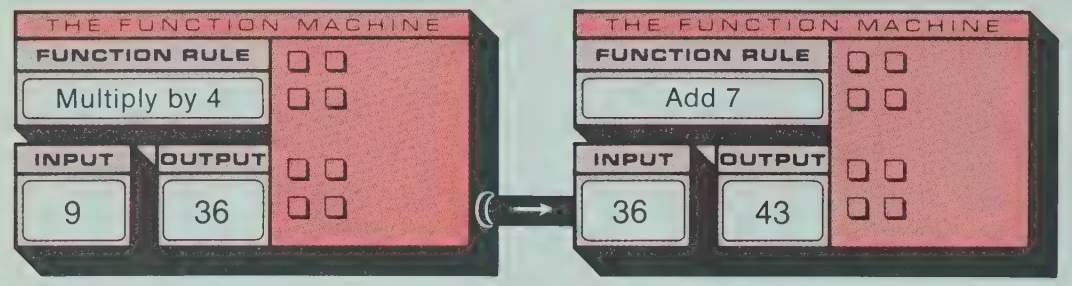
$9 \times 5 = 45 \rightarrow 4 + 5 = \boxed{9}$

What did you discover? Each digit 1 through 9 was used one time.

Combining Operations

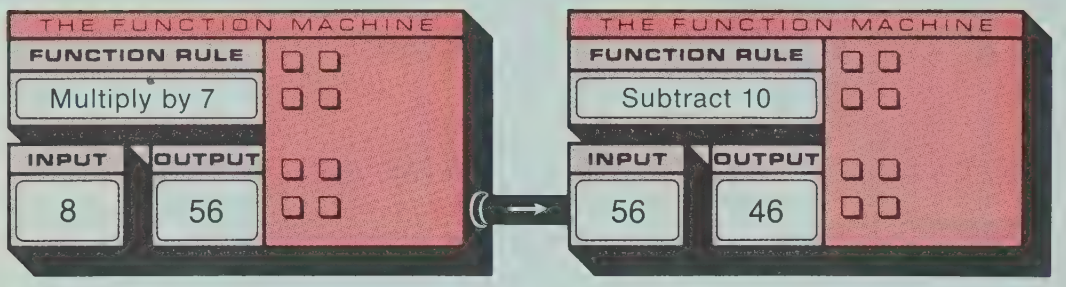
The output of the first machine becomes the input of the second machine. Give the numbers in the table or the missing rule for each exercise.

1.



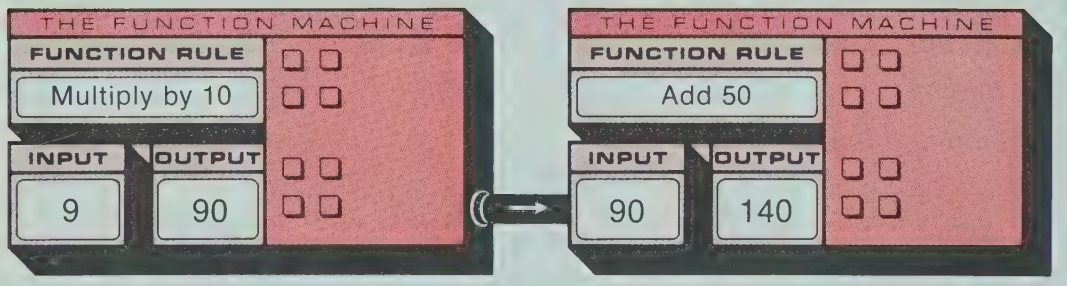
	First Input	Final Output
	9	43
A	8	39
B	7	35
C	6	31

2.



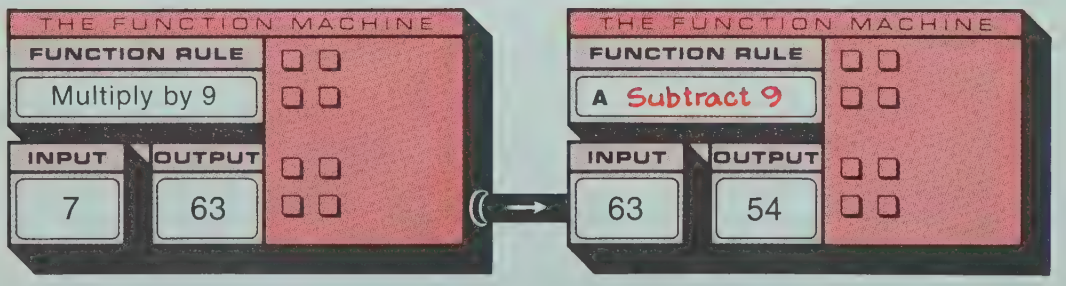
	First Input	Final Output
	8	46
A	9	53
B	10	60
C	7	39

3.



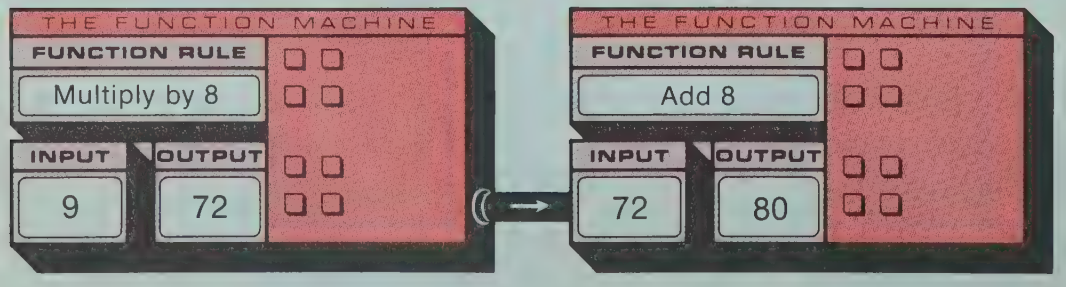
	First Input	Final Output
	9	140
A	8	130
B	7	120
C	6	110

4.



	First Input	Final Output
	7	54
	6	45
B	5	36
C	4	27

5.



	First Input	Final Output
	9	80
A	4	40
B	6	56
C	7	64

Extending the Table

1. How many of these products can you find? Write them in the table. The exercises below the table may help you.

×	5	6	7	8	9	10	11	12	13	14	15
5	25	30	35	40	45	50	55	60	65	70	75
6	30	36	42	48	54	60	66	72	78	84	90
7	35	42	49	56	63	70	77	84	91	98	105
8	40	48	56	64	72	80	88	96	104	112	120
9	45	54	63	72	81	90	99	108	117	126	135
10	50	60	70	80	90	100	110	120	130	140	150
11	55	66	77	88	99	110	121	132	143	154	165
12	60	72	84	96	108	120	132	144	156	168	180
13	65	78	91	104	117	130	143	156	169	182	195
14	70	84	98	112	126	140	154	168	182	196	210
15	75	90	105	120	135	150	165	180	195	210	225

2. First write the products in the red part of the table. Then use the red part to help you find the products in the light gray and the dark gray part.

You can find 12 × 7 by adding 6 × 7 to 6 × 7.

A What is 12 × 7? 84 B What is 7 × 12? 84

3. You can find 13 × 8 by adding 6 × 8 to 7 × 8.

A What is 13 × 8? 104 B What is 8 × 13? 104

Encourage children to use the multiplication-addition principle to find these larger products.

Writing Multiplication Story Problems

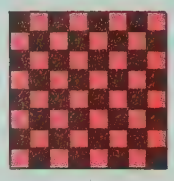
Part of a story problem or a picture suggesting a story problem is given in each exercise. Complete the story problem so that when you solve the equation you solve the problem.

1. Brad bought 6 cartons of cola. There were 8 bottles in each carton.

How many bottles of cola?

$6 \times 8 =$ 48

2. There are 8 squares in one row of a checkerboard.



There are 8 rows.

How many small squares?

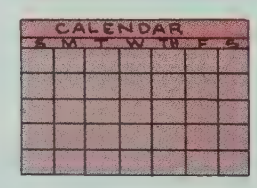
$8 \times 8 =$ 64

3. Each baseball team has 9 regular players. There are 6 teams in the league.

How many regular players in the league?

$6 \times 9 =$ 54

4. Jane works 5 hours each day.



She works for 7 days

How many hours did she work?

$7 \times 5 =$ 35

5. Have 9 boxes of crayons. There are 8 crayons in each box. How many crayons?

$9 \times 8 =$ 72

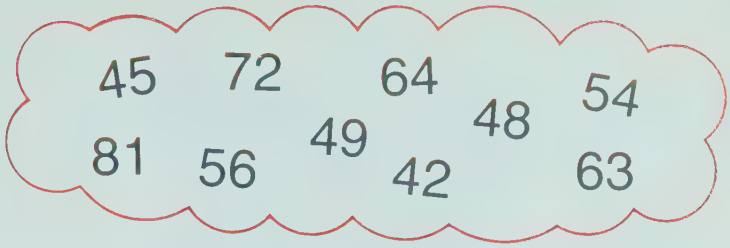
6. Answers will vary

____ $\times$ ____ $=$ ____

Make up a problem of your own and write the equation.

Larger Products

There are just 10 basic-fact products above 40.



How many basic-fact equations can you write using these numbers as products?

6 × 7 = 42
7 × 6 = 42
5 × 9 = 45
9 × 5 = 45
6 × 8 = 48
8 × 6 = 48
7 × 7 = 49
6 × 9 = 54
9 × 6 = 54
8 × 7 = 56
7 × 8 = 56
9 × 7 = 63
7 × 9 = 63
8 × 8 = 64
9 × 8 = 72
8 × 9 = 72
9 × 9 = 81

Now see if you can do these in less than 2 minutes. Remember the answers are given above.

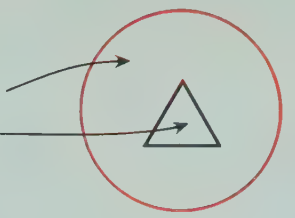
6 × 8 = 48
9 × 6 = 54
7 × 7 = 49
7 × 9 = 63
9 × 9 = 81
8 × 8 = 64
6 × 7 = 42
9 × 5 = 45
8 × 9 = 72

Encourage children to think systematically about writing the basic-fact equations. In other words using the products from smallest to largest and applying the order principle for multiplication. Thus, if 6 × 7 = 42 then 7 × 6 = 42.

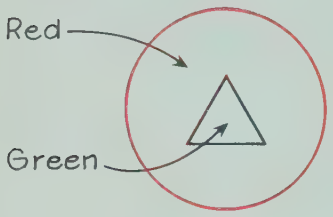
How Many Ways?

Complete each list.

1. Use 3 colors (Red, Blue, or Yellow) for this part.
Use 2 colors (Green or Brown) for this part.



How many different designs can you make? _____
Draw and color them in the space below.



(B,G)(Y,G)
(R,B)(B,B)(Y,B)

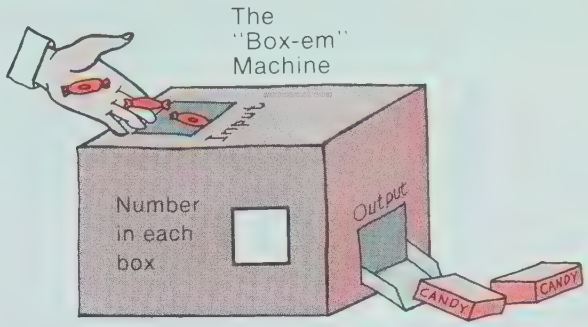
2. You are a baseball coach. You have 4 pitchers (**A**ker, **B**ar, **C**ox, and **D**unn) and 3 catchers (**R**ay, **S**tar, and **T**odd). A pitcher-catcher combination is called a battery. How many different batteries can you choose? 12 List them here: (A,R)(A,S)(A,T)(B,R)(B,S)(B,T)
(C,R)(C,S)(C,T)(D,R)(D,S)(D,T)

3. You have 5 blouses (or shirts), (Blue, Green, White, Pink, and Tan) and 3 skirts (or trousers), (Brown, Red, and Black). How many different blouse, skirt (or shirt, trouser) outfits can you choose? 15 List them here: (blue, brown)(blue,red)(blue,black)
(green,brown)(green,red)(green,black)(white,brown)
(white,red)(white,black)(pink,brown)(pink,red)
(pink,black)(tan,brown)(tan,red)(tan,black)

This lesson should help children discover that pairing and multiplication are related.

Boxes and Division

The "Box-Em" Machine puts the same number of objects in each box.



Complete this record sheet for the machine and solve the equations.

	Record for the "Box-em" Machine				
	ITEM	INPUT (number of items)	Number in each box	OUTPUT (number of boxes)	EQUATION
1.	Crayons	32	8	4	32 ÷ 8 = 4
2.	Cookies	24	6	4	24 ÷ 6 = 4
3.	Felt Pens	28	4	7	28 ÷ 4 = 7
4.	Toy Cars	35	5	7	35 ÷ 5 = 7
5.	Marbles	54	9	6	54 ÷ 9 = 6
6.	Erasers	360	9	40	360 ÷ 9 = 40
7.	Candy Bars	18	6	3	18 ÷ 3 = 6
8.	Rulers	90	10	9	90 ÷ 9 = 10
9.	Ping-Pong Balls	32	8	4	32 ÷ 4 = 8
10.	Books	20	5	4	20 ÷ 4 = 5
11.	Answers will vary				

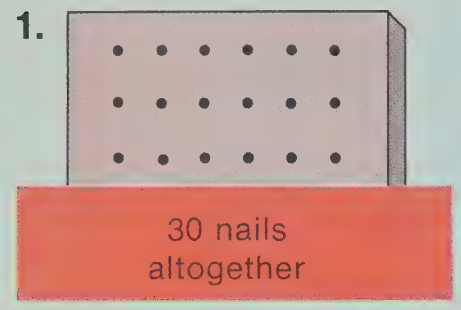
Choose an item of your own.

The Box-em Machine is a practical way of introducing division. Children's thinking may be as follows: If I put 32 crayons in the machine and there are 8 in each box, how many eights are there in 32. Some children may discover that multiplication and division are related.

Nailboard Coverups

Some of the nailboards are partly covered. Figure out how many rows of nails are on each board and write a division equation.

1.



Number of rows 5

30 ÷ 6 = 5

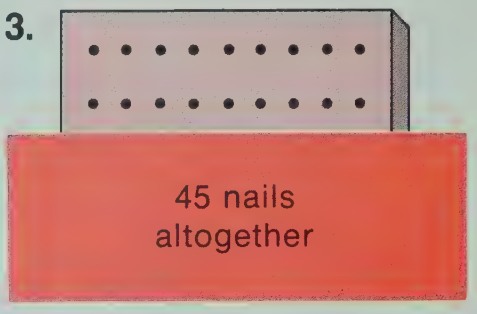
Number of nails in all. Number of nails in each row. No. of rows.

2.

Number of rows 3

21 ÷ 7 = 3

3.



Number of rows 5

45 ÷ 9 = 5

4.

Number of rows 9

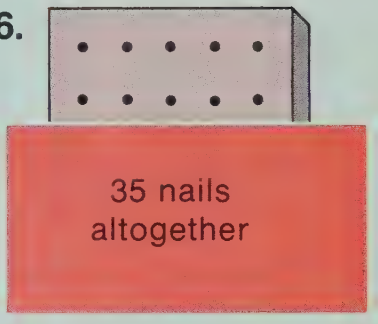
36 ÷ 4 = 9

5.

Number of rows 5

40 ÷ 8 = 5

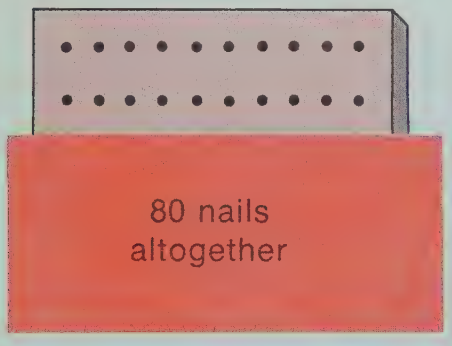
6.



Number of rows 7

35 ÷ 5 = 7

7.



Number of rows 8

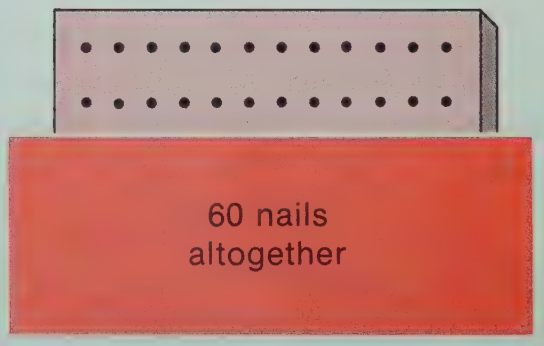
80 ÷ 10 = 8

8.

Number of rows 9

27 ÷ 3 = 9

9.



Number of rows 5

60 ÷ 12 = 5

Subtracting to Find Quotients

Complete the following by subtracting until you reach zero.
Then solve the division equation.

EXAMPLE:

108		
− 9	(1)	
99		
− 9	(2)	
90		
− 9	(3)	
81		
− 9	(4)	
72		
− 9	(5)	
63		
− 9	(6)	
54		
− 9	(7)	
45		
− 9	(8)	
36		
− 9	(9)	
27		
− 9	(10)	
18		
− 9	(11)	
9		
− 9	(12)	
0		
108 ÷ 9 =	12	

1.

72	
−12	(1)
60	
−12	(2)
48	
−12	(3)
36	
−12	(4)
24	
−12	(5)
12	
−12	(6)
0	
72 ÷ 12 =	6

2.

120	
− 15	(1)
105	
−15	(2)
90	
−15	(3)
75	
−15	(4)
60	
−15	(5)
45	
−15	(6)
30	
−15	(7)
15	
−15	(8)
0	
120 ÷ 15 =	8

3.

104	
− 13	(1)
91	
−13	(2)
78	
−13	(3)
65	
−13	(4)
52	
−13	(5)
39	
−13	(6)
26	
−13	(7)
13	
−13	(8)
0	
104 ÷ 13 =	8

4.

126	
− 18	(1)
108	
−18	(2)
90	
−18	(3)
72	
−18	(4)
54	
−18	(5)
36	
−18	(6)
18	
−18	(7)
0	
126 ÷ 18 =	7

Some children may decide to take short cuts in the subtracting process. This is highly desirable and you should encourage such activity whenever it occurs.

Finding Missing Factors

In each mini-multiplication table the products are given.
Can you find the missing factors and write them in the tables?

Some answers may vary.

1.

×	5	8
0	0	0
2	10	16

2.

×	3	7
1	3	7
4	12	28

3.

×	4	2
6	24	12
3	12	6

4.

×	2	5
3	6	15
4	8	20

5.

×	6	8
4	24	32
7	42	56

6.

×	5	9
2	10	18
9	45	81

7.

×	9	5
3	27	15
8	72	40

8.

×	4	3
6	24	18
5	20	15

9.

×	4	9
5	20	45
8	32	72

10.

×	9	2
4	36	8
7	63	14

11.

×	8	6
9	72	54
5	40	30

12.

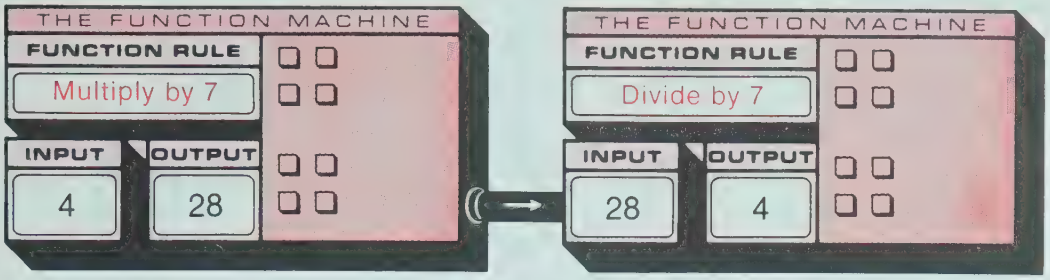
×	7	6
9	63	54
8	56	48

Observe that in finding the factors in the table the child must think about the factors of a given product or divisors and quotients.

Division and Multiplication

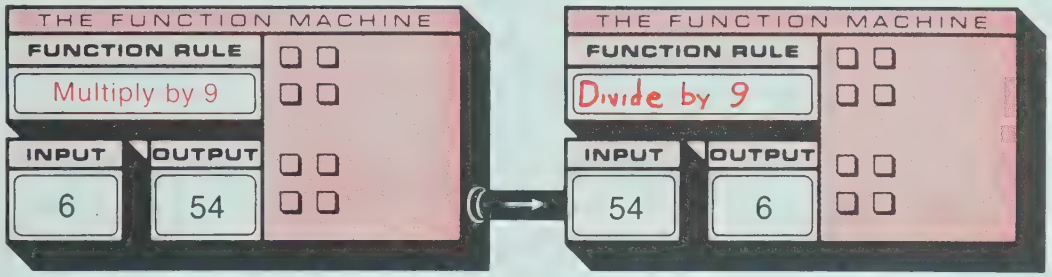
When 2 function machines are hooked together, the output of the first machine becomes the input for the second machine. Give the numbers in the table or the missing rule for each exercise.

1.



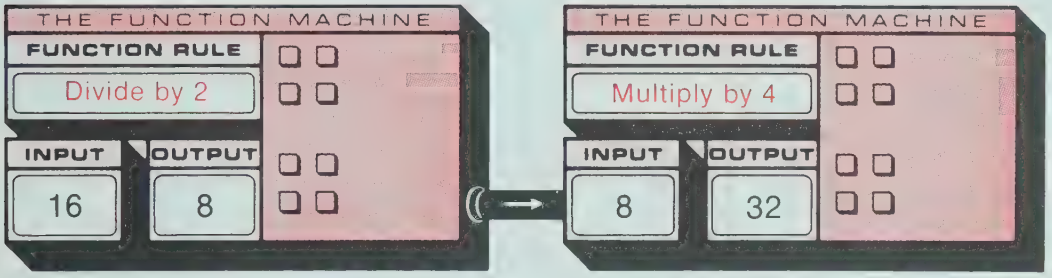
	First Input	Final Output
	4	4
A	6	6
B	9	9
C	12	12

2.



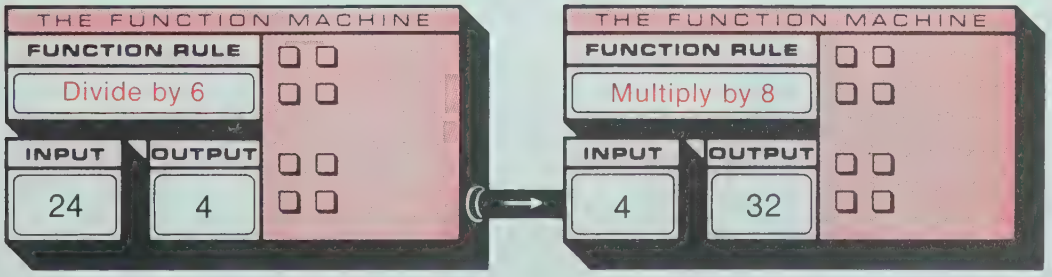
	First Input	Final Output
	6	6
	2	2
A	8	8
B	52	52

3.



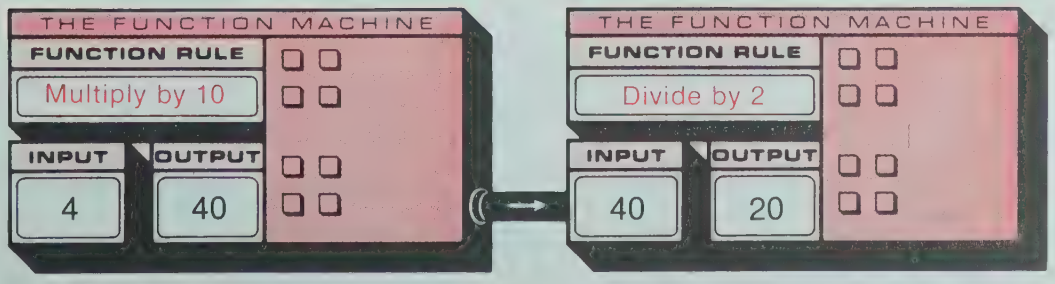
	First Input	Final Output
	16	32
A	12	24
B	14	28
C	10	20

4.



	First Input	Final Output
	24	32
A	30	40
B	48	64
C	12	16

5.



	First Input	Final Output
	4	20
A	5	25
B	6	30
C	9	45

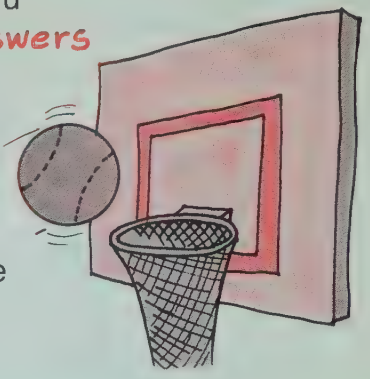
Writing Division Story Problems

Part of a story problem or a picture suggesting a story problem is given in each exercise. Complete the story problem so that when you solve the equation you have solved the story problem. Sample answers are given

1. Pam's Girl Scout troup had 56 boxes of cookies to sell. There were eight girls in her troup.

How many boxes of cookies should each girl sell?

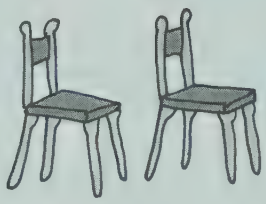
$56 \div 8 =$ 7



2. Ted's gym class wanted to play basketball. There were 35 boys in the class.

If there are 5 boys on each team, how many teams will there be?

$35 \div 5 =$ 7



3. Mrs. Good wanted to arrange the chairs for a movie in her classroom. She wanted 9 chairs in each row.

If there are 27 chairs, how many rows will there be?

$27 \div 9 =$ 3

4. Jan's family drove at a speed of 50 kilometers each hour on a trip to Bigtown.

How long did it take them to travel 250 kilometers?

$250 \div 50 =$ 5

5. Bob bought 24 bottles of pop. If each carton contains 6 bottles, how many cartons did he buy?

$24 \div 6 =$ 4

6. answers will vary

____ ÷ ____ =

Make up a division problem of your own and write the equation.

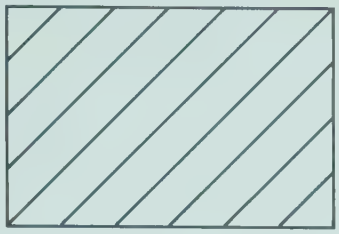
Each child may complete the story problems differently. However, the important thing is that the equation is correct for their story problem.

Making Parallel Line Designs

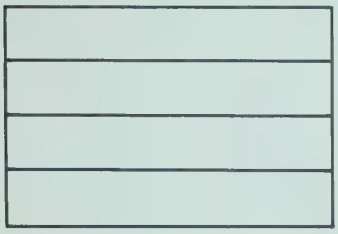
In the space at the bottom, draw **parallel lines** following steps 1 to 4.

Step 1: Draw these.

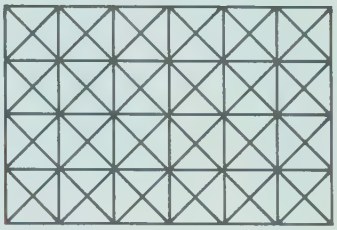
Step 2: Draw these.



Step 3: Draw these.

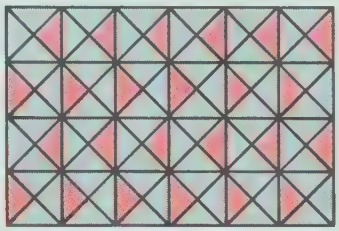


Step 4: Draw these.



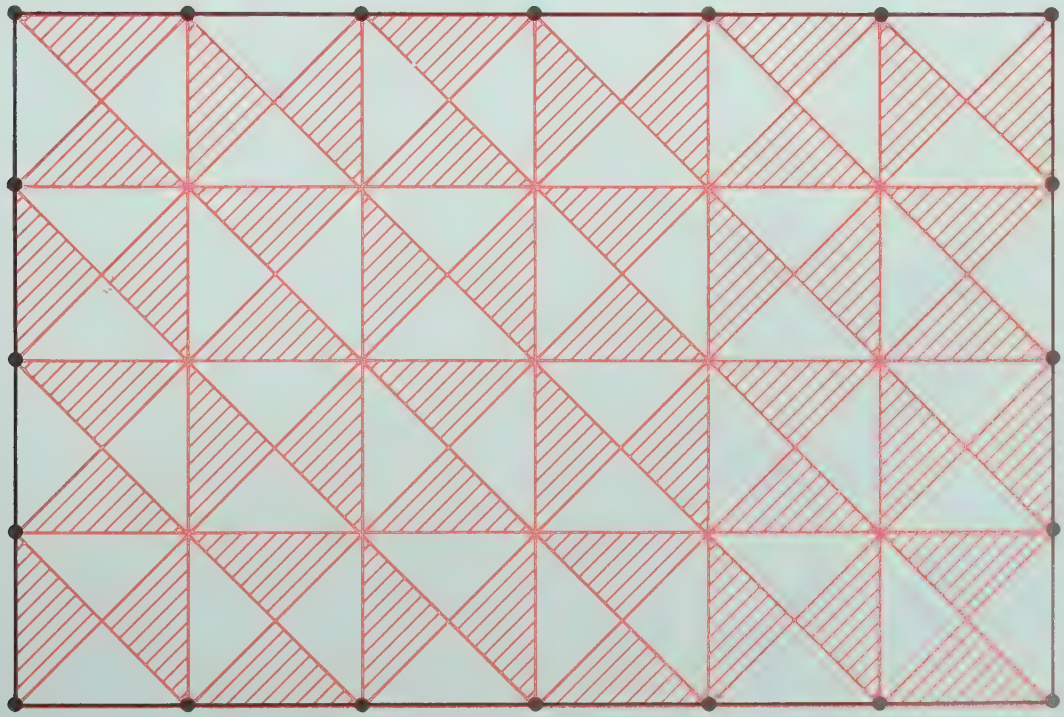
Your design should look like this.

This figure shows one possible design using red and gray. . . . . .



Color a design of your own on the grid you made below.

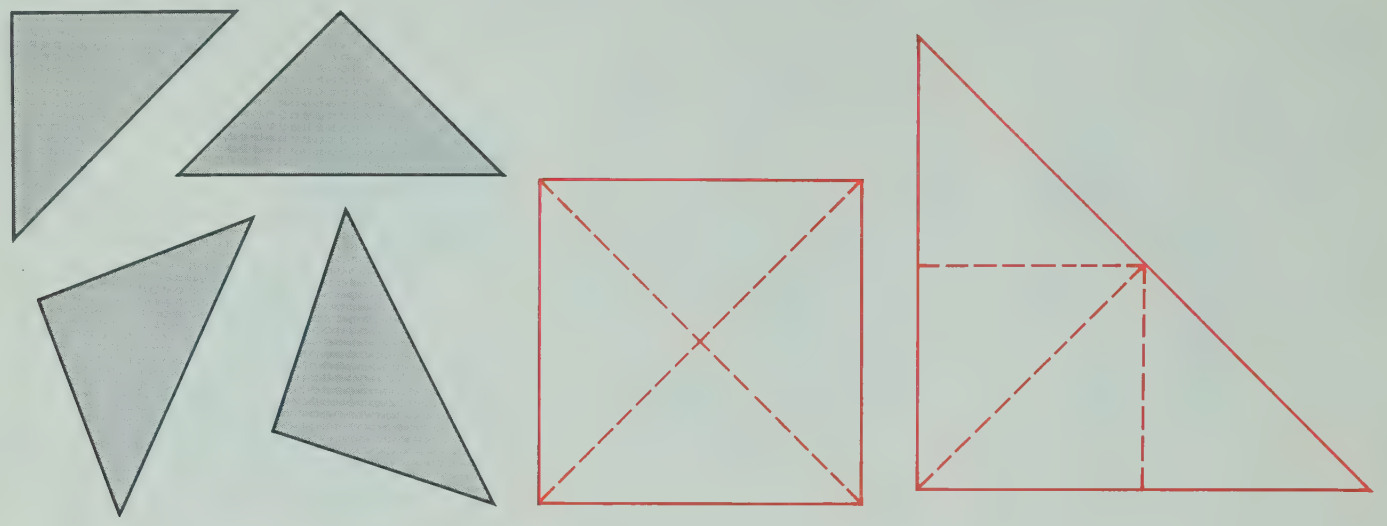
Sample Design :



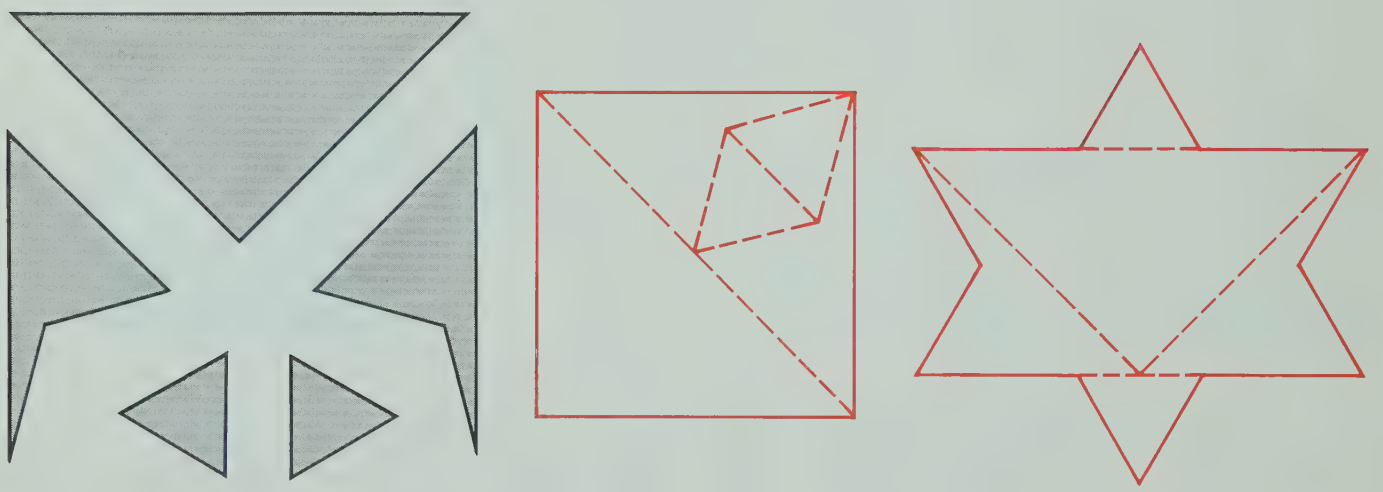
Puzzle Pieces

Cut out the gray pieces. Use all the pieces and fit them inside the red outline(s).

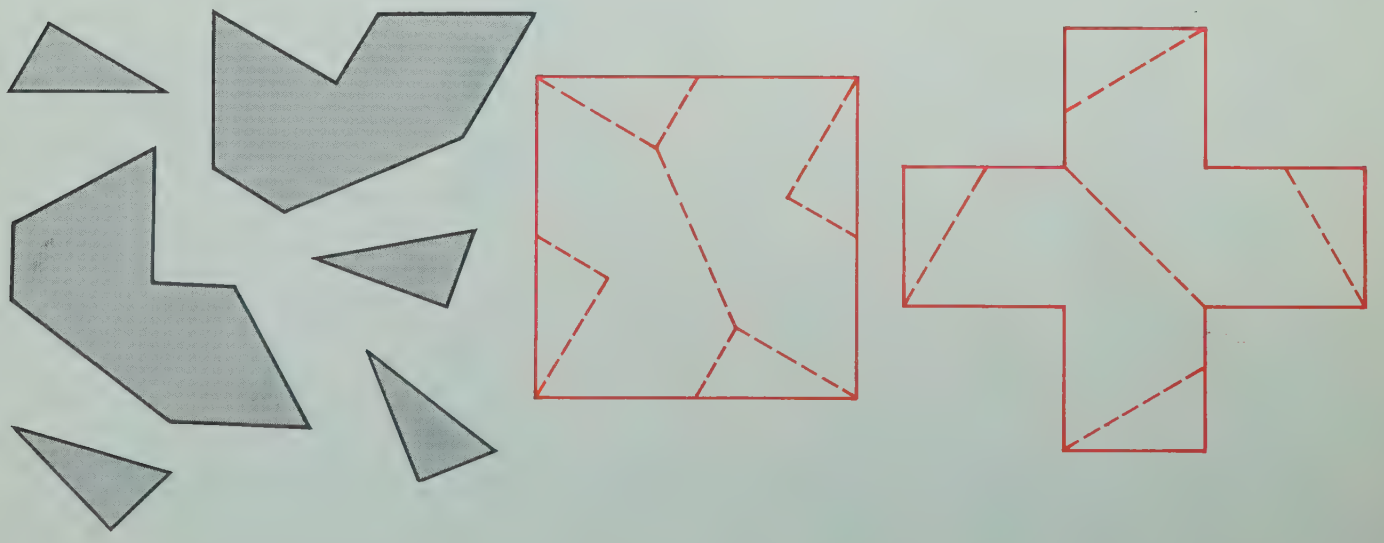
1.



2.

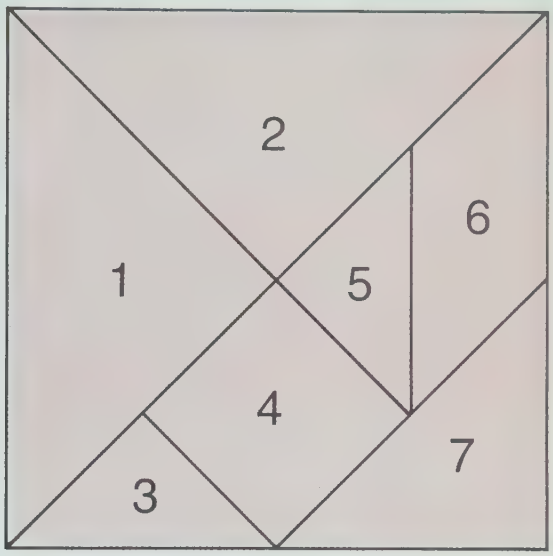


3.

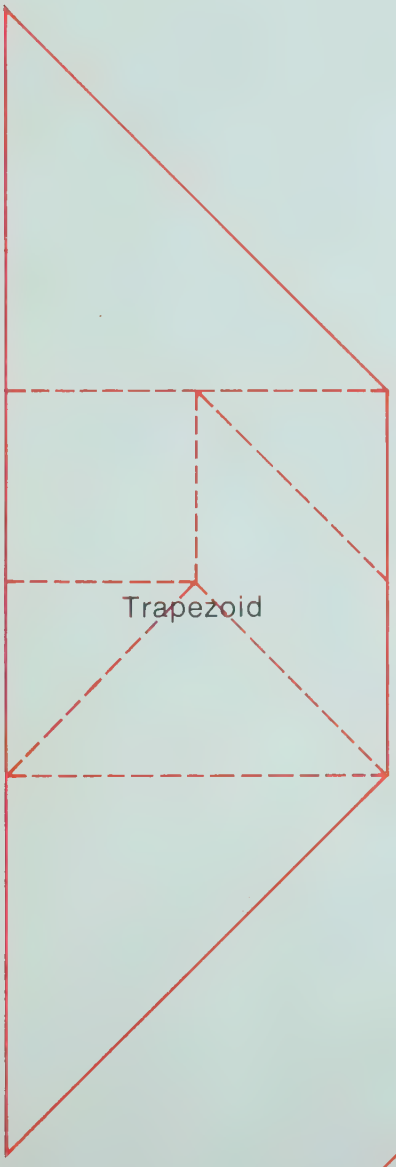


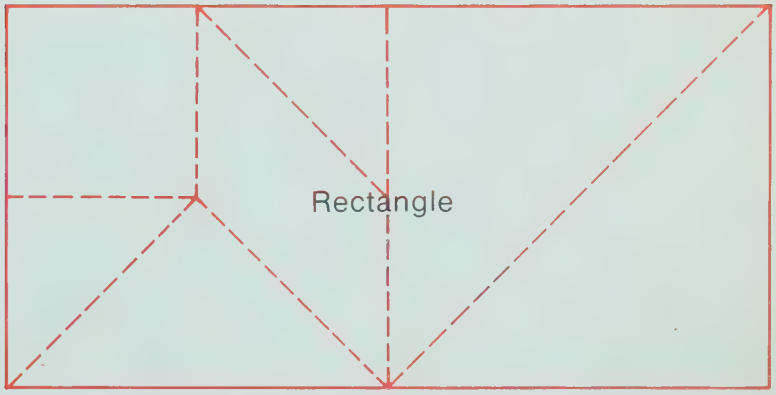
The Tangram Puzzle

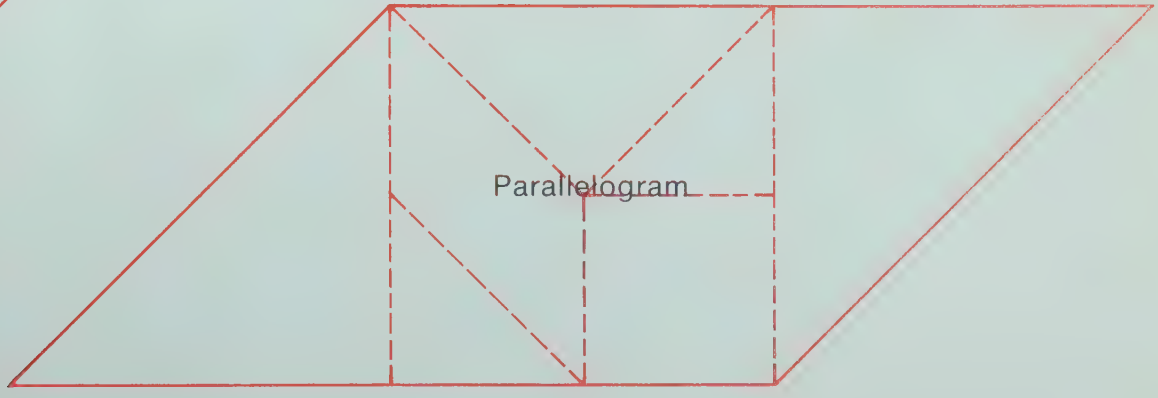
Trace and cut out the 7 **Tangram** pieces. Then figure out how to fit all the pieces, without overlapping, in each red outline below.



TANGRAM PIECES

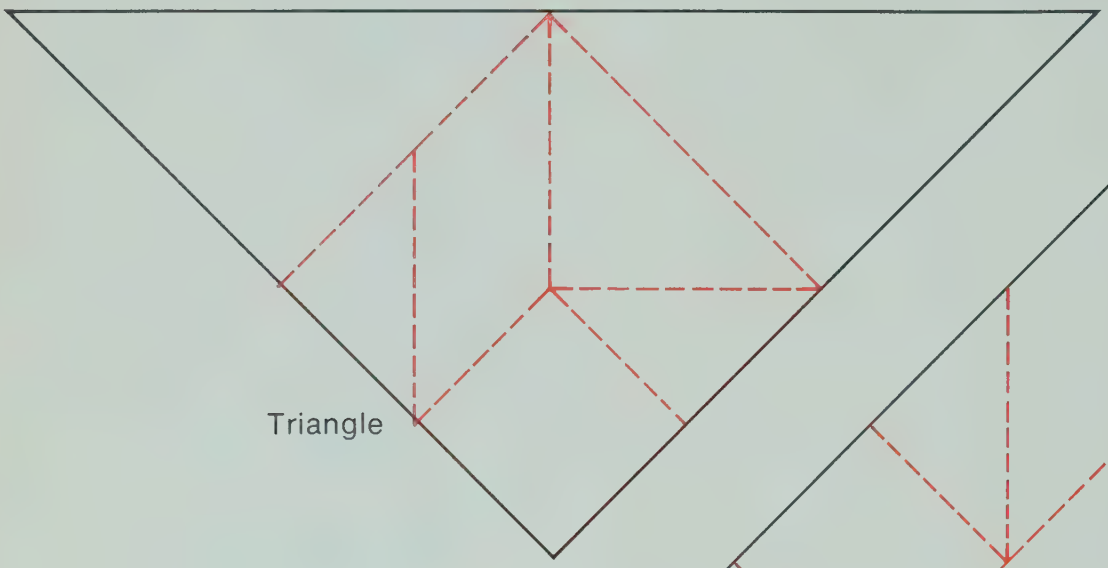




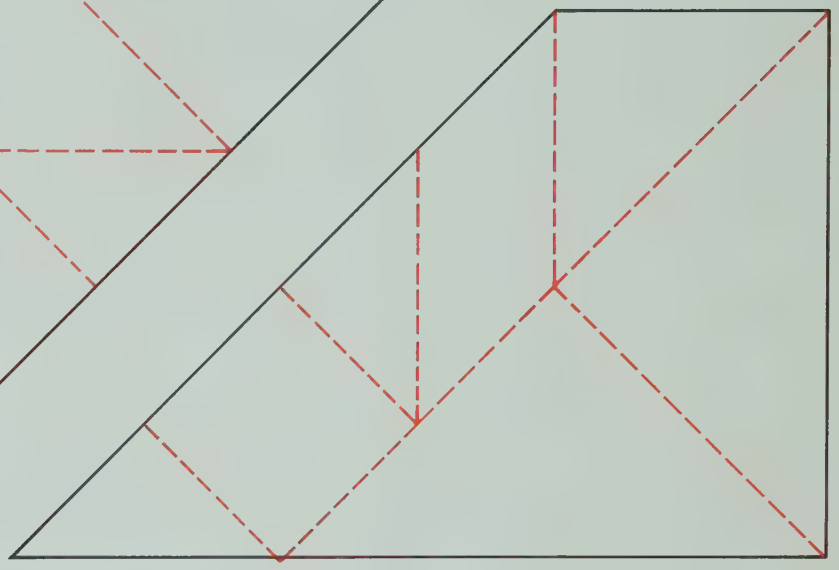


Tangram Polygons

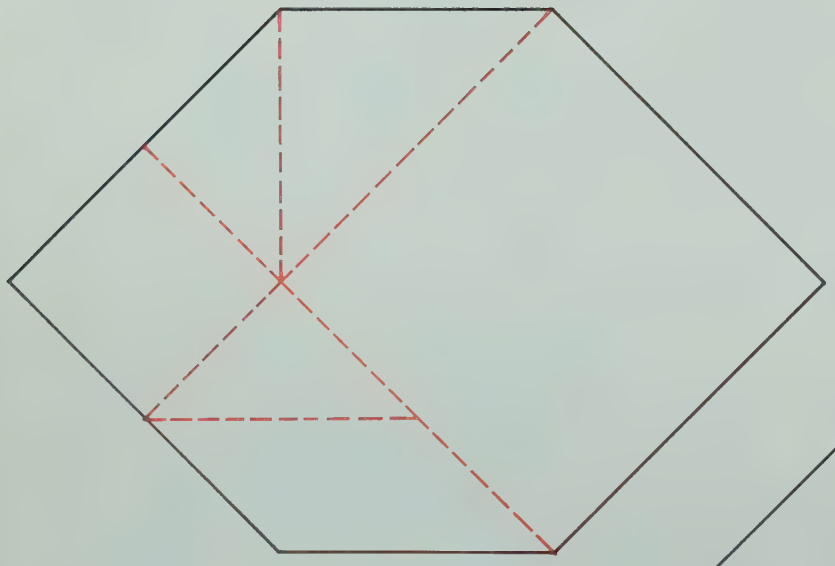
Use all your **Tangram** pieces. Draw lines to show how to completely fill, each of the **polygon** outlines with no overlapping.



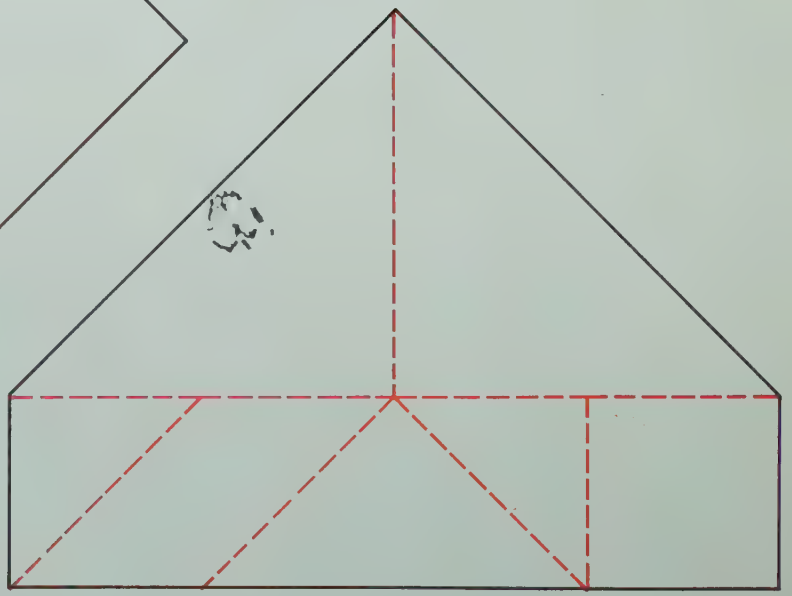
Triangle



Quadrilateral



Hexagon

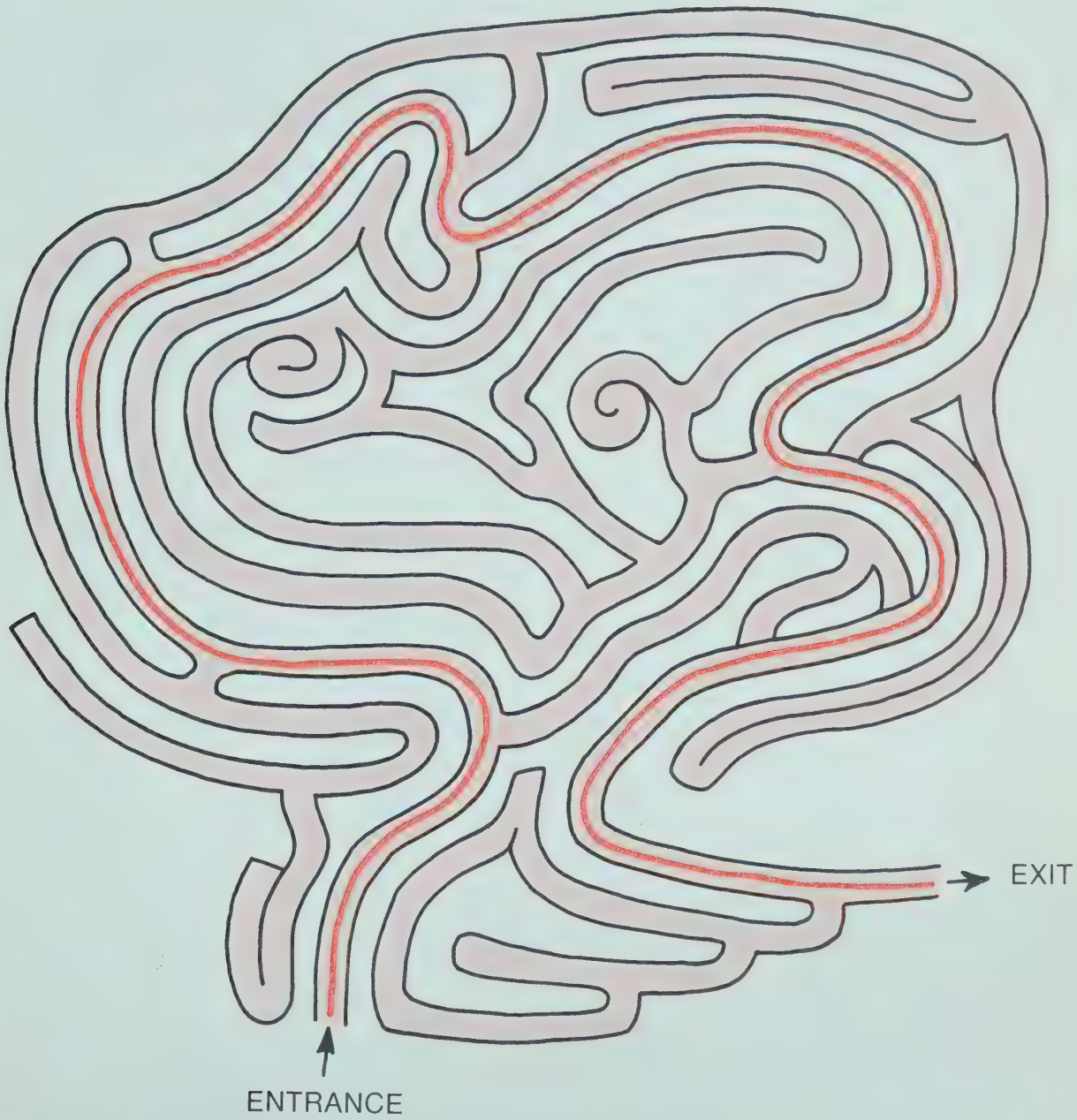


Pentagon

After finishing this page you might suggest that children try to make some figures on their own.

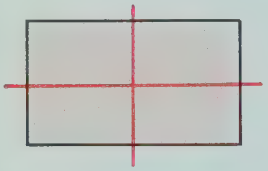
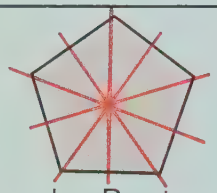
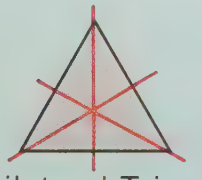
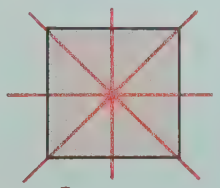
Solving a Maze

Can you find a path from the ENTRANCE to the EXIT?
Show it with your pencil.



Symmetry

Complete this table. Find as many different lines of **symmetry** as you can. Draw them on the figure.

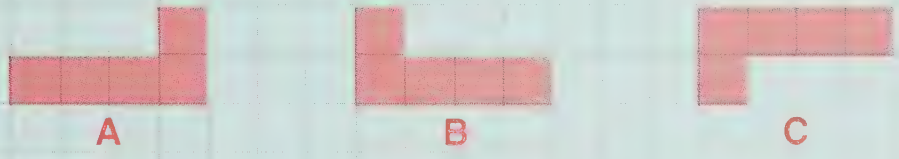
	Figure	Number of Sides	Number of Vertices (Corners)	Number of Lines of Symmetry
1.	 Isosceles Triangle	3	3	1
2.	 Rectangle	4	4	2
3.	 Regular Pentagon	5	5	5
4.	 Equilateral Triangle	3	3	3
5.	 Regular Hexagon	6	6	6
6.	 Square	4	4	4

"5 Square Figures"

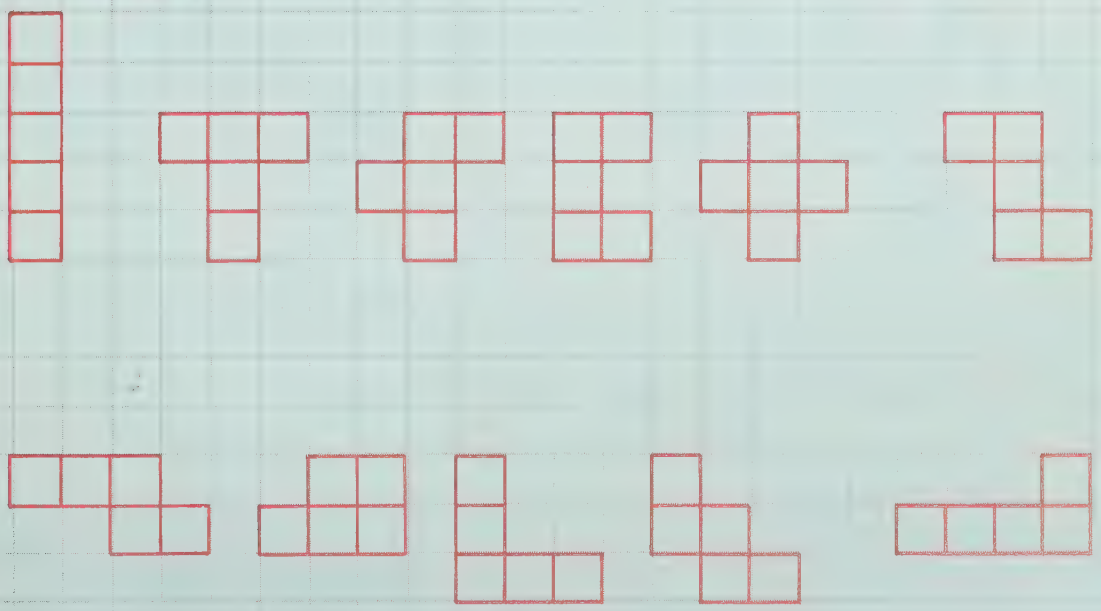
There are 12 *different* figures which can be made by coloring 5 squares on graph paper. (Each square must touch at least one other square along a complete side. Since both **B** and **C** can be flipped or turned to look like **A**, we do not call them different from **A**.)

Color as many of the 12 different "5 square figures" as you can. If any of the figures have lines of symmetry, show them.

These are all considered the same.



This is a starter for you.



To help children understand when two figures are considered the same have them think of cutting out a figure and see if it will fit in any way on the other figures. If not, then the figures are considered different.

Number Puzzles

Can you solve these number puzzles?

1. When you multiply the smallest even number (not zero) by itself and add 1, you get me.

Who am I? 5

$2 \times 2 + 1 = 5$

2. I'm the product of two odd numbers less than 10. I am a "teenager."

Who am I ? 15

$3 \times 5 = 15$

3. I'm the product of two even numbers less than 10. I can "vote," but I'm under 30.

Who Am I? 24

$4 \times 6 = 24$

4. I'm the product of an even and an odd number less than 10 that are "close friends." I'm over 30, but under 50.

Who am I? 42

$6 \times 7 = 42$

5. If you multiply the largest odd number less than 10 by itself and add the largest odd number less than 20 to this product you get me.

Who am I? 100

$9 \times 9 + 19 = 100$

6. I'm twice the product of the largest even number less than 10 times the smallest even number greater than 0.

Who am I? 32

$2 \times 8 \times 2 = 2 \times 16$

$= 32$

7. I'm the product of two of the smallest 2-digit even numbers. My last digit is zero.

Who am I? 120

$10 \times 12 = 120$

8. If you multiply one of the very smallest even numbers by itself enough times, you'll get me. I'm over 50 and under 100.

Who am I? 64

$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Even and Odd Numbers

Write one of the even digits 0, 2, 4, 6, or 8 on each E

Write one of the odd digits 1, 3, 5, 7, or 9 on each O

Be sure your problems are correct. There is usually more than one correct answer.

EXAMPLE:

$$\begin{array}{r} E\ O \\ +\ \ E \\ \hline O\ O \end{array}$$

Possible Answer:

$$\begin{array}{r} 4\ 3 \\ +\ \ 6 \\ \hline 4\ 9 \end{array}$$

Can you find a different answer for the above example? Answers will vary.

1. $+\ E$ **2.** $+\ E$ **3.** $+\ O$ **4.** $+\ O$ **5.** $+\ O$ **6.** $+\ O$

7. $+\ E$ **8.** $+\ E$ **9.** $+\ O$ **10.** $+\ E$ **11.** $+\ E$ **12.** $+\ O$

13. $+\ E\ E$ **14.** $+\ E\ E$ **15.** $+\ E\ O$ **16.** $+\ E\ O$ **17.** $+\ E\ O$ **18.** $+\ E\ O$

19. $\times\ E$ **20.** $\times\ O$ **21.** $\times\ O$ **22.** $\times\ O$ **23.** $\times\ E$ **24.** $\times\ E$

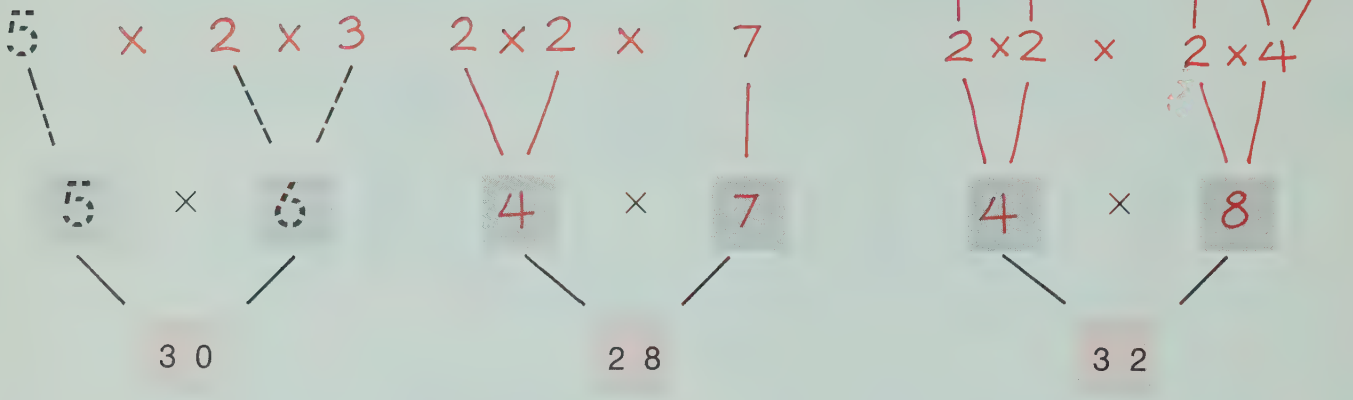
25. $\times\ E$ **26.** $\times\ O$ **27.** $\times\ O$ **28.** $\times\ E$ **29.** $\times\ E$ **30.** $\times\ O$

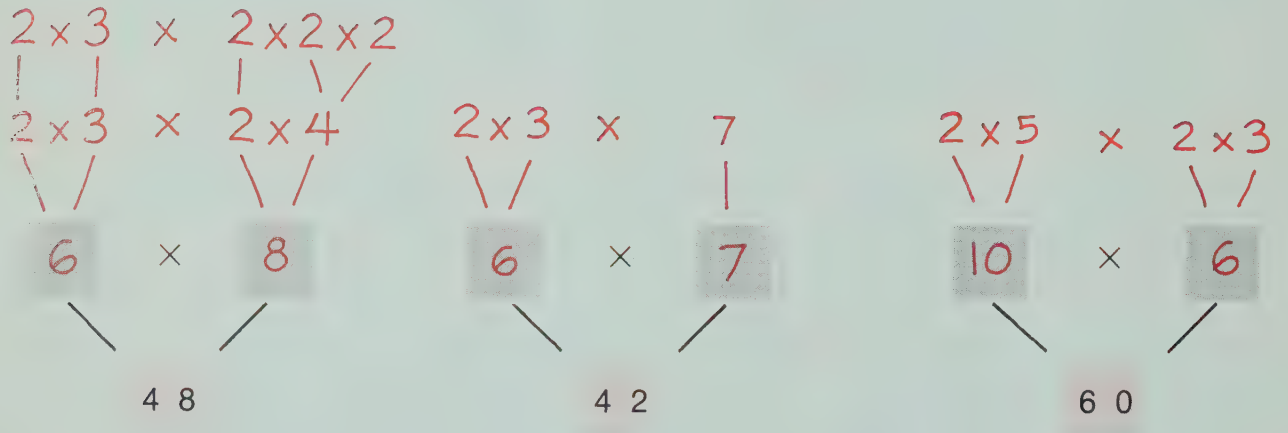
More than one answer is possible for many of the exercises on this page.

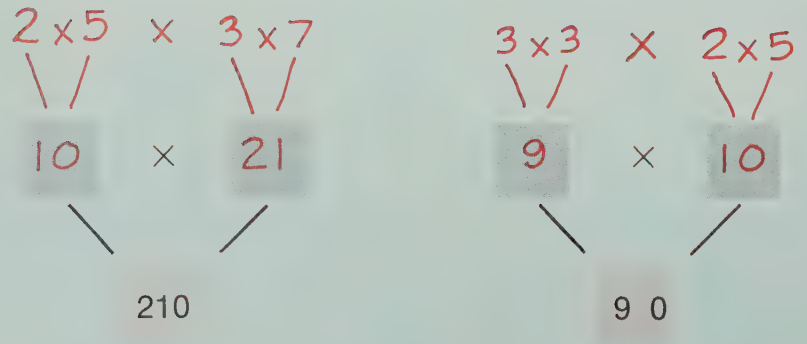
More Factor Trees

Make each "factor tree" grow as much as you can.

Remember that 1 is not used in factor trees.







Use another sheet of paper and make some more factor trees of your own.

Sorting Out Primes

1. Follow these rules:

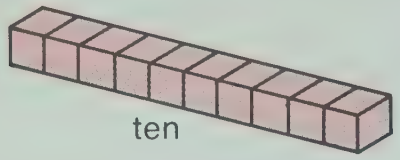
- **A** Color 1 gray. It is not a prime number.
- **B** 2 is a prime. Do not color 2. A multiple of 2 is not prime. Color all other multiples of 2 red.
- **C** 3 is a prime. Do not color 3. A multiple of 3 is not prime. Color all other multiples of 3 blue.
- **D** 5 is a prime. Do not color 5. A multiple of 5 is not prime. Color all other multiples of 5 yellow.
- **E** 7 is a prime. Do not color 7. A multiple of 7 is not prime. Color all other multiples of 7 green.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

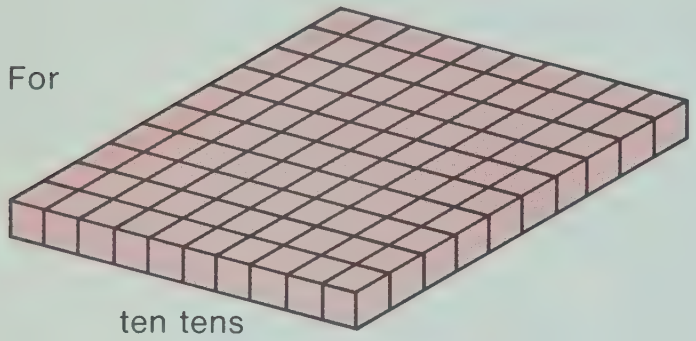
2. The numbers in the squares not colored are the **Prime Numbers** less than 100. How many prime numbers less than 100 are there? 25

After sorting out the prime numbers you might want to expand this lesson by relating it to the previous page. Ask the children what they notice about the numbers in the top row of the factor trees.

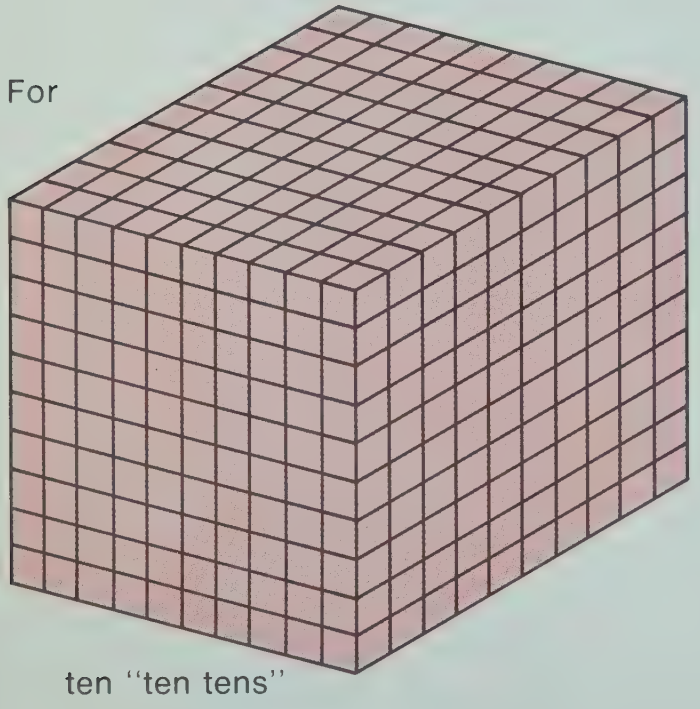
Tens, Tens, and Tens

For 

We write 10^1 (Read "ten")

For 

We write 10^2
(Read "ten squared")

For 

We write 10^3
(Read "ten cubed")

How many?

1. $10^2 =$ 100
2. $10^3 =$ 1000
3. $2 \times 10^1 =$ 20
4. $3 \times 10^2 =$ 300
5. $4 \times 10^3 =$ 4000
6. $23 \times 10^1 =$ 230
7. $42 \times 10^2 =$ 4200
8. $9 \times 10^3 =$ 9000
9. $387 \times 10^1 =$ 3870
10. $25 \times 10^3 =$ 25,000
11. $423 \times 10^2 =$ 42,300
12. $99 \times 10^3 =$ 99,000

For children to solve these problems it is important for them to observe that 10^1 is just another name for "ten". 10^2 is just another name for 100 and 10^3 is the same as 1000. It is not intended that exponents be taught at this level but rather children have an idea of what it means to square or cube a number.

Solving Inequalities

Write some of the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 that could be written in the ____ to make a true statement.

Then write the largest possible such number in the ____.

1. 0, 1, 2, 3, 4 ____ 4 ____ × 10 < 43

2. 0, 1, 2, 3, 4, 5, 6, 7, 8 ____ 8 ____ × 10 < 87

3. 0, 1, 2, 3, 4, 5, 6 ____ 6 ____ × 20 < 123

4. 0, 1, 2, 3 ____ 3 ____ × 30 < 96

5. 0, 1, 2, 3, 4, 5, 6, 7, 8 ____ 8 ____ × 40 < 329

6. 0, 1, 2, 3, 4 ____ 4 ____ × 50 < 213

7. 0, 1, 2, 3, 4, 5, 6, 7 ____ 7 ____ × 60 < 432

8. 0, 1, 2, 3, 4, 5, 6, 7, 8 ____ 8 ____ × 70 < 575

9. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ____ 9 ____ × 80 < 735

10. 0, 1, 2, 3, 4 ____ 4 ____ × 90 < 372

Notice that in finding the largest number which will make the inequality true the child is finding the first quotient digit in the problem involving dividing the second number by the first.

Using the Multiplication-Addition Principle

Start with the "2 row" of the table and fill in both gray squares.
Then find the sum of the products for this row and complete the problem below the table.

Do the same for each of the other rows in the table.

×	10	20	30	1	2	3	4	Product Sum
2		40				6		46
3	30						12	42
4			120		8			128
5		100				15		115
6			180				24	204
7	70				14			84
8		160		8				168
9			270				36	306

"2 row"	"3 row"	"4 row"	"5 row"	"6 row"	"7 row"	"8 row"	"9 row"
2 3	1 4	3 2	2 3	3 4	1 2	2 1	3 4
×2	×3	×4	×5	×6	×7	×8	×9
46	42	128	115	204	84	168	306

On another sheet of paper write 8 more problems than can be solved using other squares in the table. Fill in these squares to solve the problems.

By using the multiplication-addition principle, multiplication with one 2-digit factor becomes a simple extension of the basic facts. In each case children will rely on previously known facts to arrive at new products.

Magic With Operations

Choose numbers and follow the directions.
Why do you think the operations are called magic? The final number is the same as the original.

Sample answer given:

1.

	Choose your numbers. ↓			Work space
Choose a two-digit number.	53	47		
Double the digit in the tens place.	10	8		
Add 5.	15	13		
Multiply the result by 5.	75	65		
Add the digit in the ones place of the original number.	78	72		
Subtract 25.	53	47		

Sample answer given:

2.

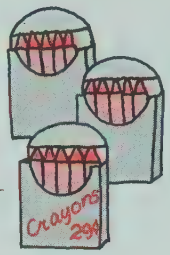
	Choose your numbers. ↓			Work space
Choose a number.	17	15		
Multiply by 5.	85	75		
Add 7.	92	82		
Multiply this sum by 5.	460	410		
Add 8.	468	418		
Multiply this sum by 4.	1872	1672		
Cross off the last 2 digits.	18	16		
Subtract 1 from the result.	17	15		

Creating Story Problems

Complete a story problem for each picture that can be solved by solving the equation. Then solve the problem.

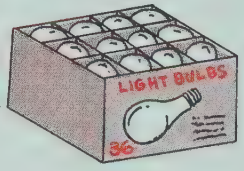
Sample story problems

1. Each box of crayons costs 29¢. How much would 3 boxes cost?



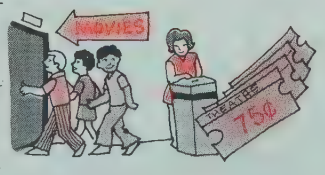
$3 \times 29 = 87$

2. There are 36 light bulbs in each box. How many bulbs in 5 boxes?



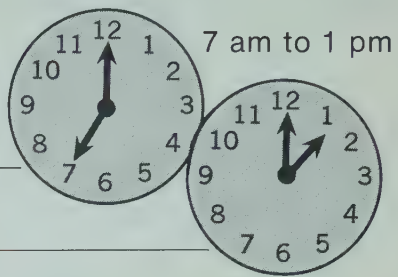
$5 \times 36 = 180$

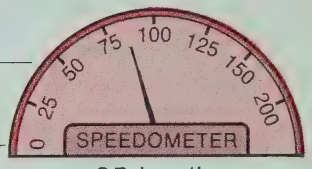
3. A ticket to the movie costs 75¢. How much will 6 tickets cost?



$6 \times 75 = 450$ or $\$4.50$

4. Drove 85 kilometers per hour for 6 hours. Traveled how far altogether?





85 km/hr

$6 \times 85 = 510$

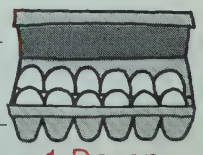
5. There are 24 hours in 1 day. How many hours are in 7 days?

CALENDAR						
S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

1 day = 24 hours

$7 \times 24 = 168$

6. There are 12 eggs (1 dozen) in each box. How many eggs in 8 boxes?



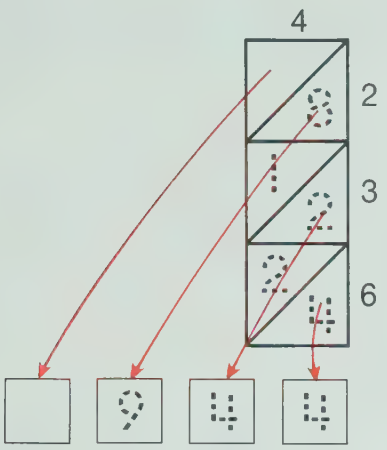
$8 \times 12 = 96$

Note that in exercise 2 the two clocks are to indicate an elapsed time of six hours while the speedometer is to indicate a speed of 85 km/hr.

A New Way to Multiply

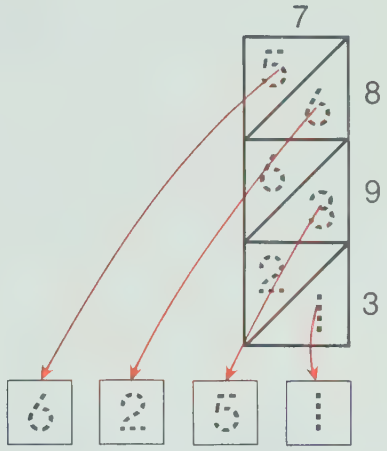
Study and complete the examples. Then use this method to find each product.

EXAMPLES:



Problem:

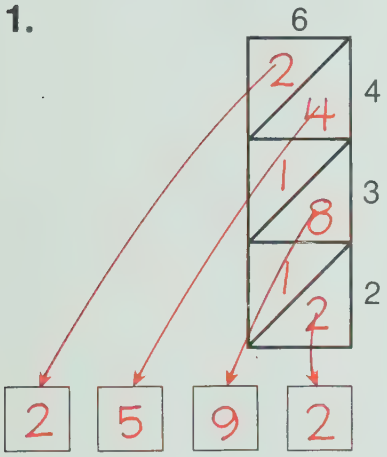
$$\begin{array}{r} 236 \\ \times\ 4 \\ \hline 944 \end{array}$$



Problem:

$$\begin{array}{r} 893 \\ \times\ 7 \\ \hline 6251 \end{array}$$

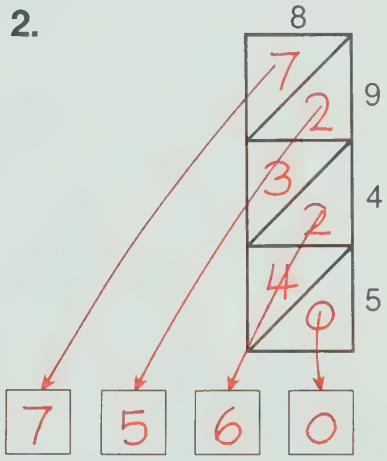
1.



Problem:

$$\begin{array}{r} 432 \\ \times\ 6 \\ \hline 2592 \end{array}$$

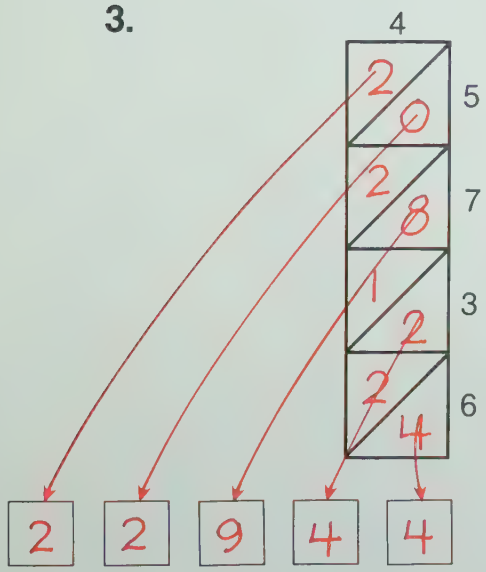
2.



Problem:

$$\begin{array}{r} 945 \\ \times\ 8 \\ \hline 7560 \end{array}$$

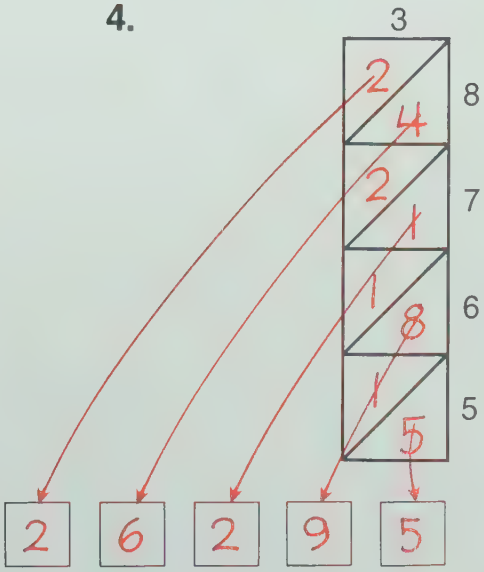
3.



Problem:

$$\begin{array}{r} 5736 \\ \times\ 4 \\ \hline 22,944 \end{array}$$

4.



Problem:

$$\begin{array}{r} 8765 \\ \times\ 3 \\ \hline 26,295 \end{array}$$

Make up some of your own on another sheet of paper.

The two diagrams at the top of the page may be sufficient to explain the method involved. However, you may need to show the children how they can write the product of the single digit numbers and then find the sums indicated by the red arrows.

Some Product Surprises

1. Find the products.

A $142857 \times 1 = 142857$

B $142857 \times 2 = 285714$

C $142857 \times 3 = 428571$

D $142857 \times 4 = 571428$

E $142857 \times 5 = 714285$

F $142857 \times 6 = 857142$

What did you discover about these problems?

The numerals 142857 are in the product of each problem.

2. Can you guess this product?

A $142857 \times 7 = 999999$

B Did you guess correctly? _____

3. Find the products.

A $123456789 \times 9 = 111,111,111$

B $37037 \times 3 = 111,111$

C $37037037 \times 3 = 111,111,111$

What did you discover about these products? All of the digits are 1.

Estimating Products

Find the products on the left and on the right of the "shaded" problem. Then check (✓) the product you think is the closest estimate to the "shaded" product.

Find the "shaded" product. Did you check (✓) the best estimate?

1.

70 × 4 = 280	73 × 4 = 292	80 × 4 = 320

2.

40 × 9 = 360	47 × 9 = 423	50 × 9 = 450

3.

60 × 8 = 480	62 × 8 = 496	70 × 8 = 560

4.

20 × 7 = 140	29 × 7 = 203	30 × 7 = 210

5.

50 × 6 = 300	58 × 6 = 348	60 × 6 = 360

6.

80 × 5 = 400	84 × 5 = 420	90 × 5 = 450

7.

300 × 4 = 1200	312 × 4 = 1248	400 × 4 = 1600

8.

600 × 3 = 1800	694 × 3 = 2082	700 × 3 = 2100

9.

800 × 6 = 4800	842 × 6 = 5052	900 × 6 = 5400

10.

5000 × 3 = 15,000	5142 × 3 = 15,426	6000 × 3 = 18,000

11.

2000 × 7 = 14,000	2864 × 7 = 20,048	3000 × 7 = 21,000

12.

8000 × 2 = 16,000	8672 × 2 = 17,344	9000 × 2 = 18,000

Children may need some assistance in extending the multiplication algorithm to three and four digit numerals.

Finding Larger Products

First find the product on the right and then on the left.
Then use these products to find the "middle product."

1.

Left	Middle	Right
68 × 2 = 136	68 × 23 = 204	68 × 3 = 204

136 × 10 = 1360

1360 + 204 = 1564

2.

Left	Middle	Right
54 × 2 = 108	54 × 27 = 378	54 × 7 = 378

108 × 10 = 1080

1080 + 378 = 1458

3.

Left	Middle	Right
38 × 4 = 152	38 × 42 = 76	38 × 2 = 76

152 × 10 = 1520

1520 + 76 = 1596

4.

Left	Middle	Right
83 × 3 = 249	83 × 37 = 581	83 × 7 = 581

249 × 10 = 2490

2490 + 581 = 3071

5.

Left	Middle	Right
78 × 5 = 390	78 × 53 = 234	78 × 3 = 234

390 × 10 = 3900

3900 + 234 = 4134

6.

Left	Middle	Right
97 × 2 = 194	97 × 26 = 582	97 × 6 = 582

194 × 10 = 1940

1940 + 582 = 2522

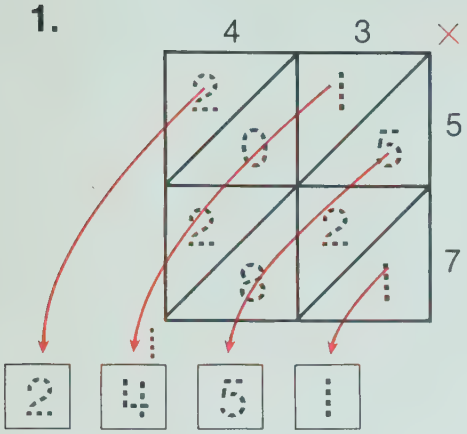
Try some more of your own.

Help the child see that since the 2 is in the ten's place (in the example) we need to multiply the left product by 10.

Special Multiplying

Study the example. Then find the products and write the completed multiplication problem.

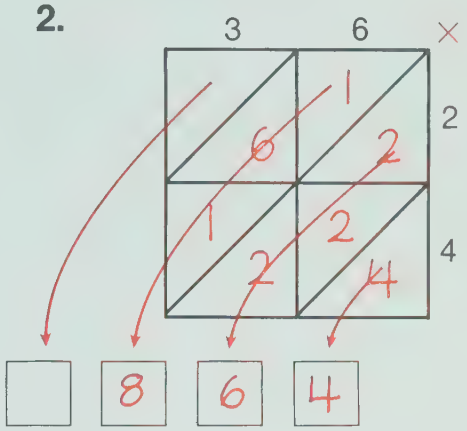
1.



Problem:

$$\begin{array}{r} 43 \\ \times\ 57 \\ \hline 2451 \end{array}$$

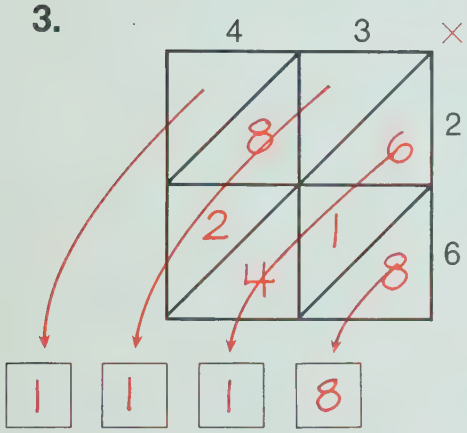
2.



Problem:

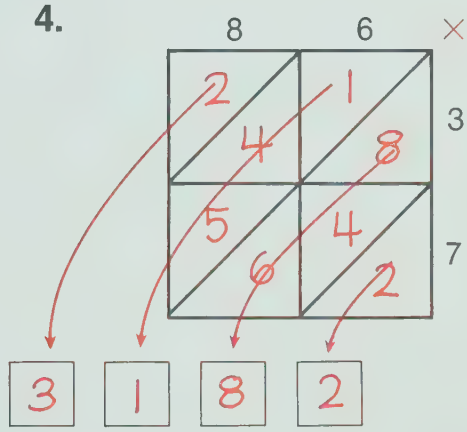
$$\begin{array}{r} 36 \\ \times 24 \\ \hline 864 \end{array}$$

3.



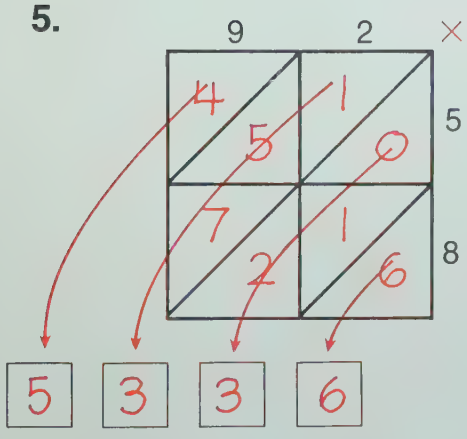
$$\begin{array}{r} 43 \\ \times\ 26 \\ \hline 1118 \end{array}$$

4.



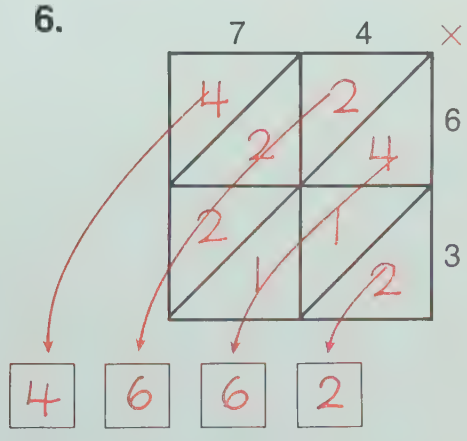
$$\begin{array}{r} 86 \\ \times 37 \\ \hline 3182 \end{array}$$

5.



$$\begin{array}{r} 92 \\ \times\ 58 \\ \hline 5336 \end{array}$$

6.



$$\begin{array}{r} 74 \\ \times 63 \\ \hline 4662 \end{array}$$

On a separate sheet of paper do 5 more of your own.

Children should study the example carefully before proceeding to the remaining exercises. Note that in the example 10 tens or 100 is carried over into the hundreds place. The dashed numeral 1 is placed there as a reminder.

For each polygon connect the points in order:

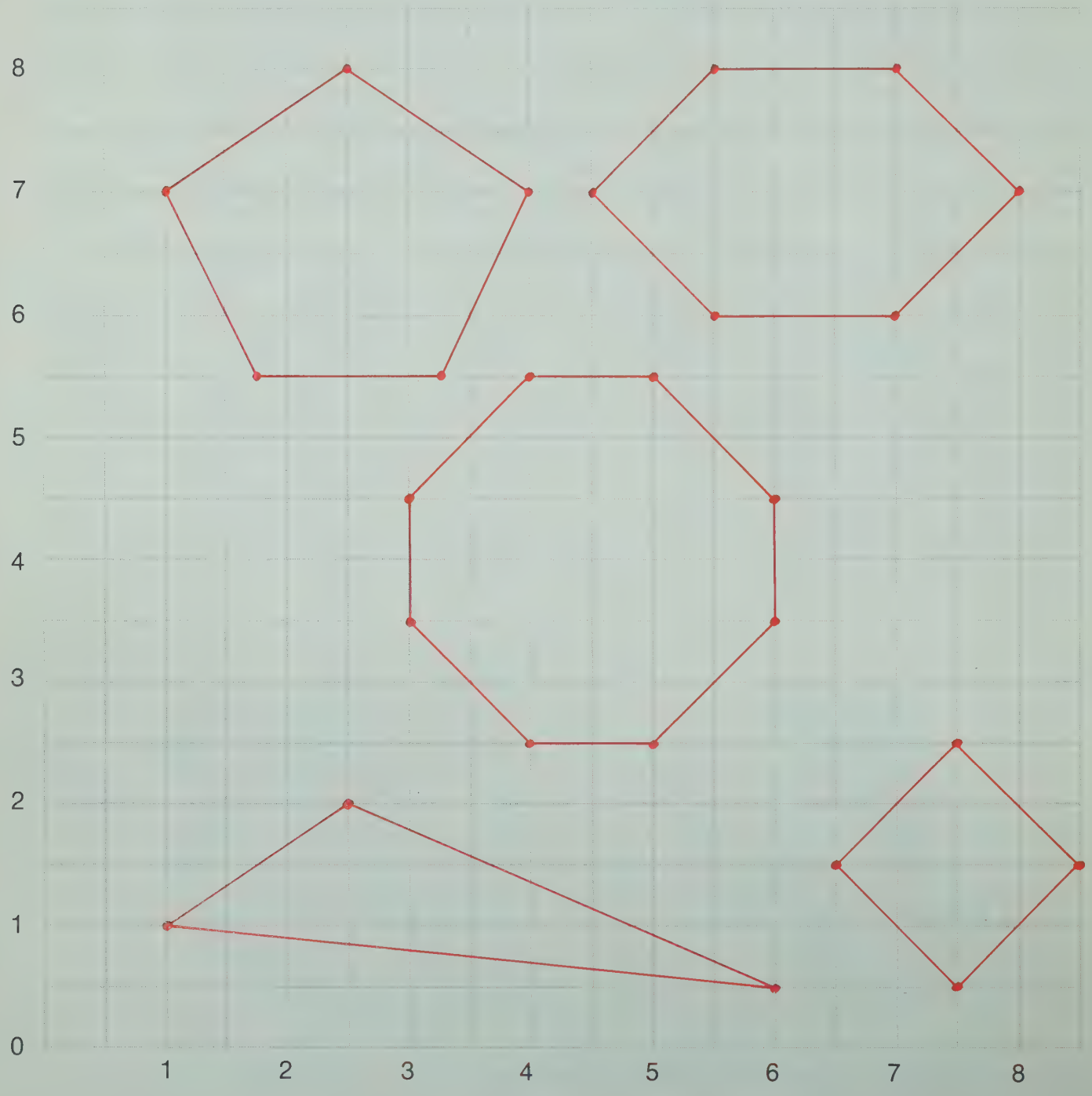
A square: $(6\frac{1}{2}, 1\frac{1}{2})$, $(7\frac{1}{2}, 2\frac{1}{2})$, $(8\frac{1}{2}, 1\frac{1}{2})$, $(7\frac{1}{2}, \frac{1}{2})$

A triangle: $(1, 1)$, $(2\frac{1}{2}, 2)$, $(6, \frac{1}{2})$

A pentagon: $(1, 7)$, $(2\frac{1}{2}, 8)$ $(4, 7)$, $(3\frac{1}{4}, 5\frac{1}{2})$, $(1\frac{3}{4}, 5\frac{1}{2})$

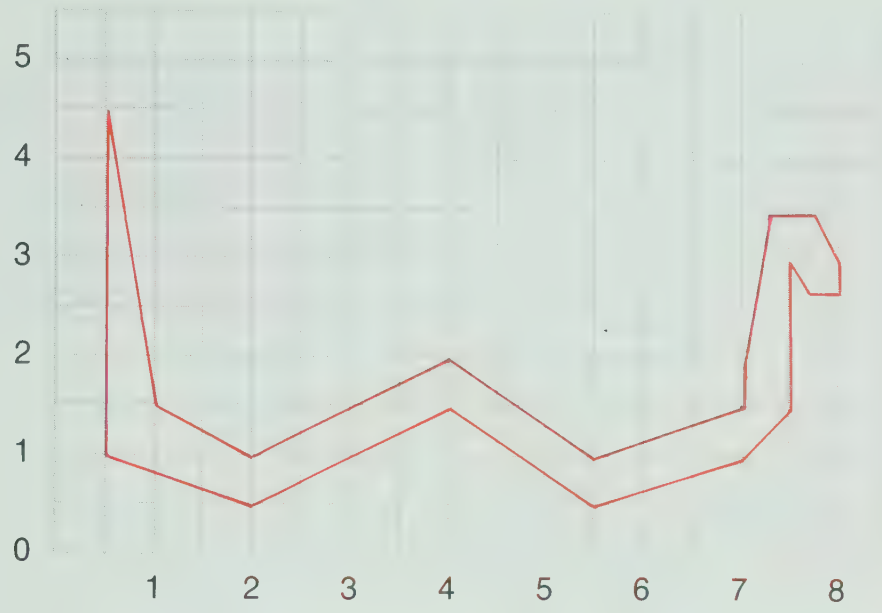
A hexagon: $(4\frac{1}{2}, 7)$, $(5\frac{1}{2}, 8)$, $(7, 8)$, $(8, 7)$, $(7, 6)$, $(5\frac{1}{2}, 6)$

An octagon: $(3, 4\frac{1}{2})$, $(4, 5\frac{1}{2})$, $(5, 5\frac{1}{2})$, $(6, 4\frac{1}{2})$, $(6, 3\frac{1}{2})$, $(5, 2\frac{1}{2})$, $(4, 2\frac{1}{2})$, $(3, 3\frac{1}{2})$



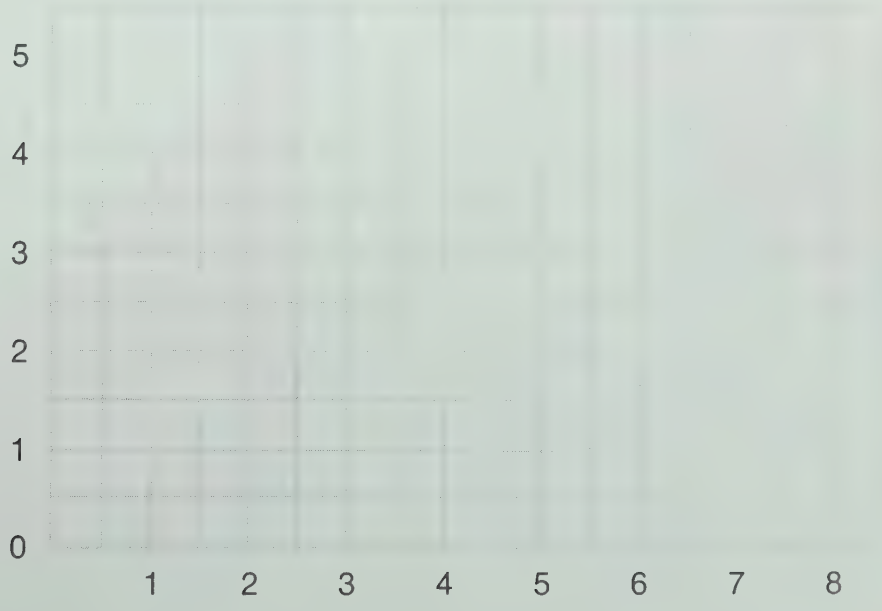
Graphing and Creating Point Pictures

1. Graph this point picture by connecting these points in order:
$(\frac{1}{2}, 4\frac{1}{2})$, $(1, 1\frac{1}{2})$, $(2, 1)$ $(4, 2)$, $(5\frac{1}{2}, 1)$, $(7, 1\frac{1}{2})$ $(7, 2)$, $(7\frac{1}{4}, 3\frac{1}{2})$, $(7\frac{3}{4}, 3\frac{1}{2})$
$(8, 3)$, $(8, 2\frac{3}{4})$, $(7\frac{3}{4}, 2\frac{3}{4})$, $(7\frac{1}{2}, 3)$, $(7\frac{1}{2}, 1\frac{1}{2})$, $(7, 1)$, $(5\frac{1}{2}, \frac{1}{2})$, $(4, 1\frac{1}{2})$, $(2, \frac{1}{2})$,
$(1, 1)$, $(\frac{1}{2}, 4\frac{1}{2})$.



2. Make a point picture of your own and give the coordinates that someone could use to graph it. _____

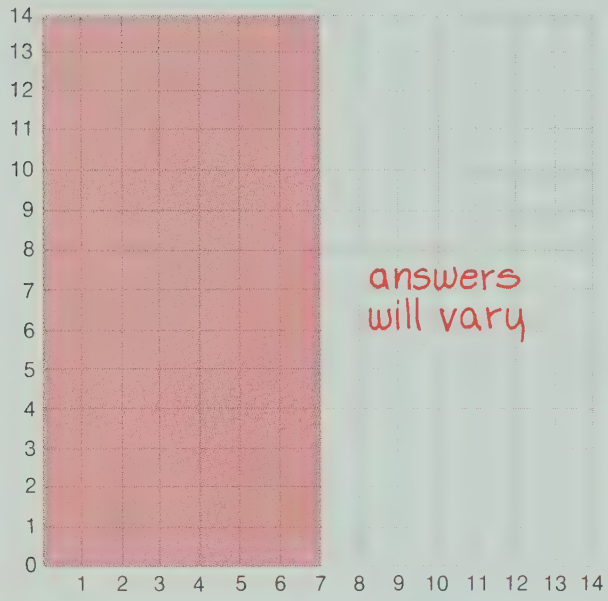
_____



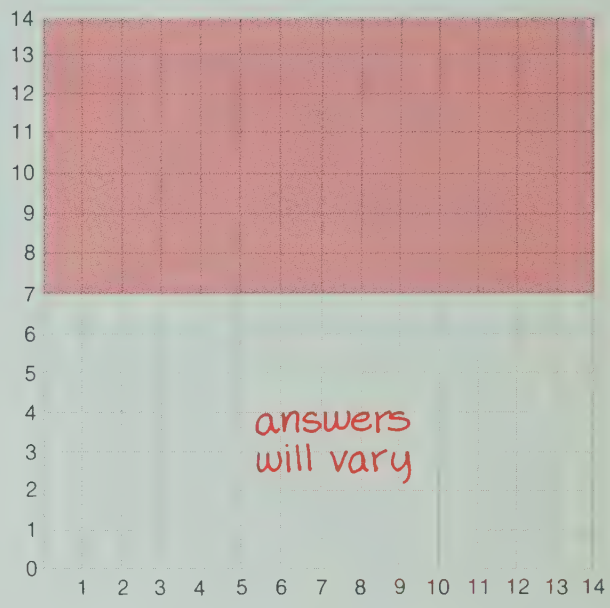
Making Symmetrical Figures

Connect points to make half of a symmetrical figure on the shaded side of the line of symmetry. Give your figures to a classmate to complete.

1.

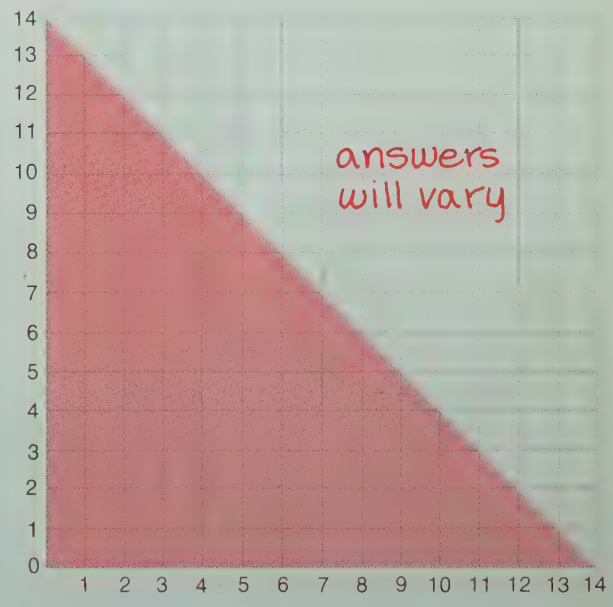


2.



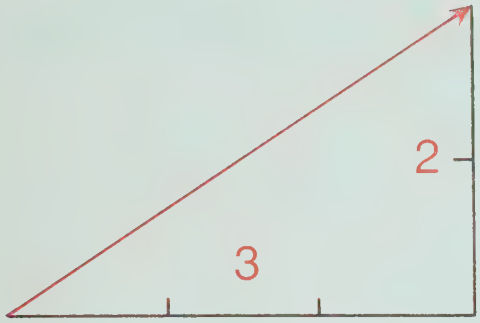
3.

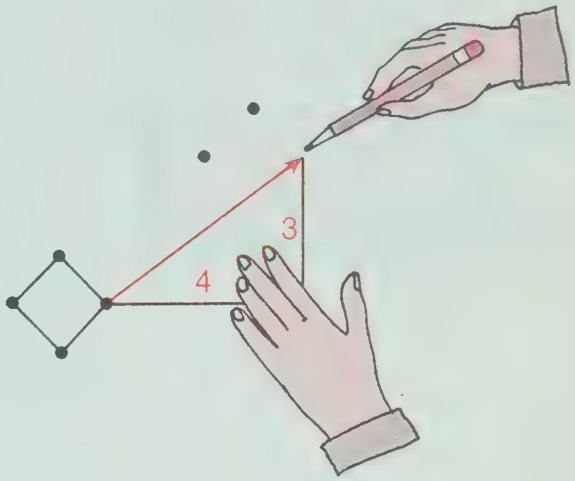
4.

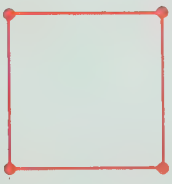


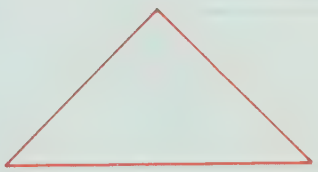
Using a Point Slider

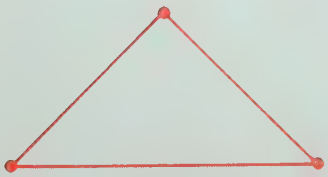
The picture on the right shows an "over 4, up 3 **point slider**" being used to "move" the square to a new position. Trace the "over 3, up 2 **point slider**" on the left, cut it out, and use it to "move" each figure below to a new position.









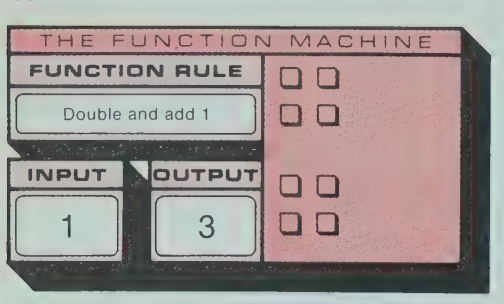


As an extension for this have the children find a different point slider of their own choosing and move other figures that they have drawn.

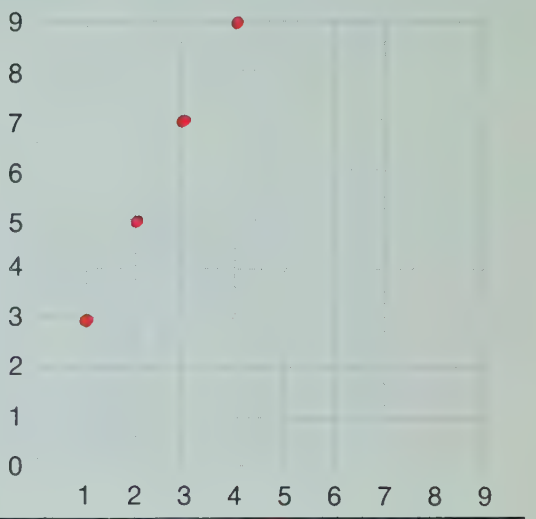
Graphing Functions

This function machine makes an input-output card each time it operates.
Show at least 4 cards for each machine and graph the point for each card.

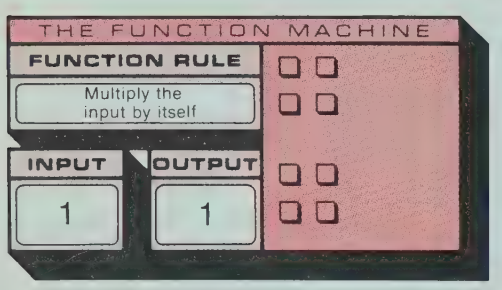
1.



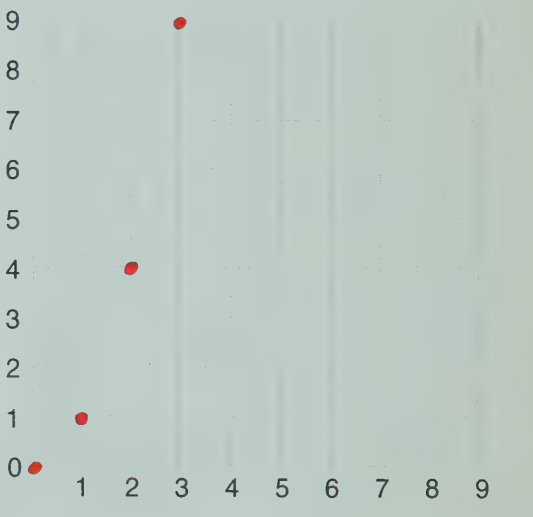
	input	output
	(1,	3)
A	(2,	5)
B	(3,	7)
C	(4,	9)



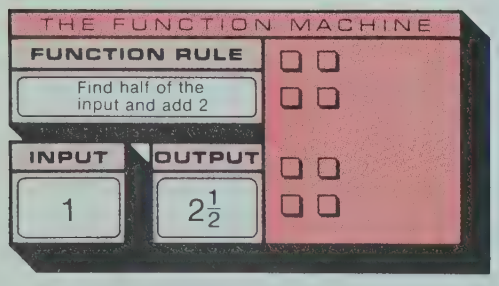
2.



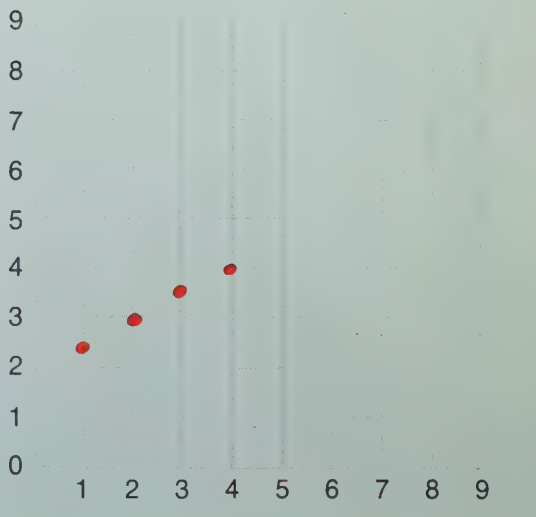
	input	output
A	(0,	0)
B	(1,	1)
C	(2,	4)
D	(3,	9)



3.



	input	output
	(1,	$2\frac{1}{2}$)
A	(2,	3)
B	(3,	$3\frac{1}{2}$)
C	(4,	4)

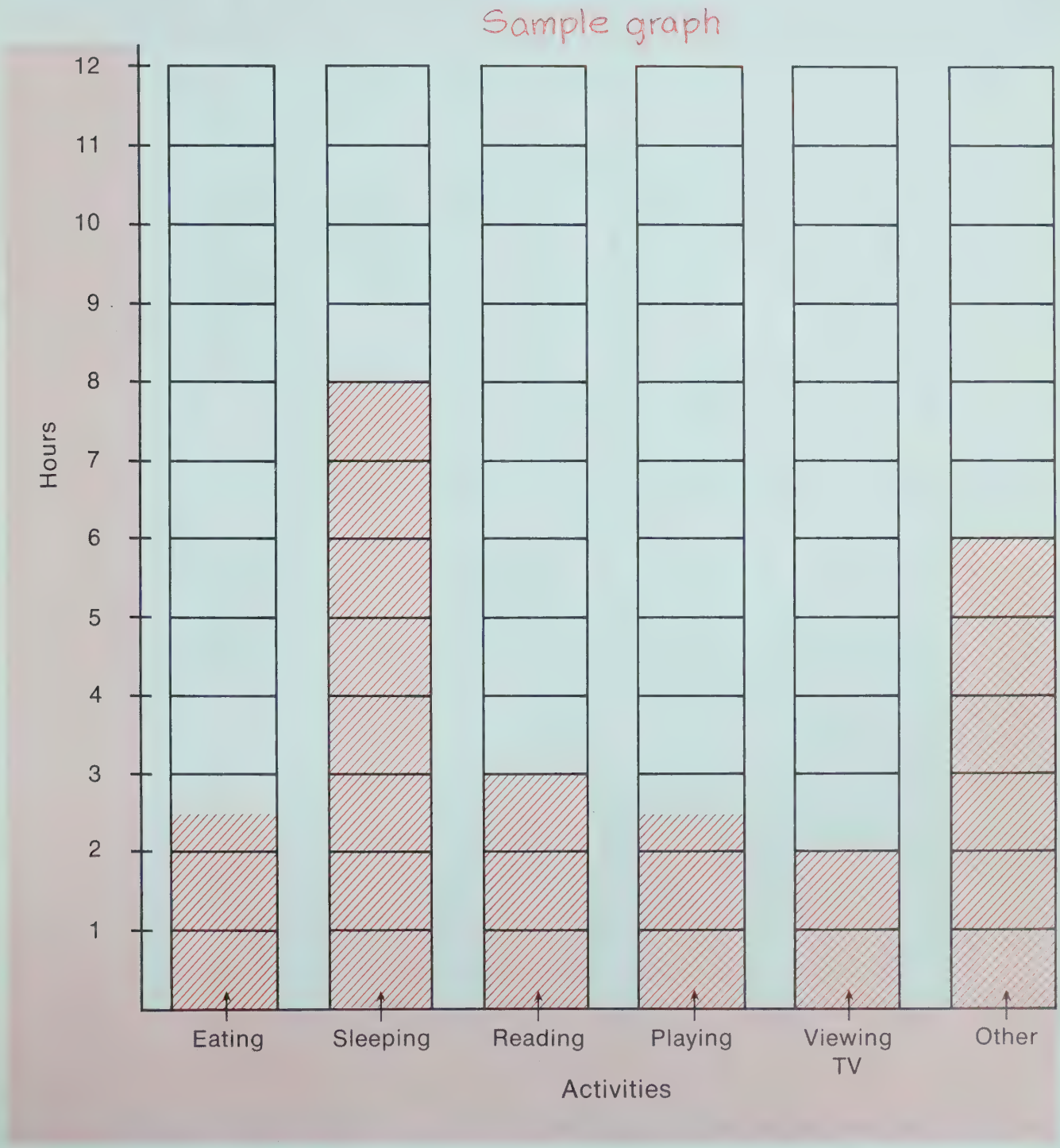


Making a Bar Graph

Decide how much time you spend during a 24-hour day on each of the activities.

Then color a square or part of a square in the graph below for each hour or part of an hour you spend on each activity.

Your completed graph should show how you spend your time in an ordinary 24-hour day.

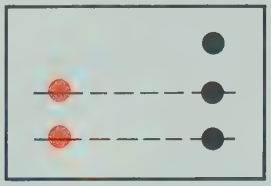


Adding Negative Numbers

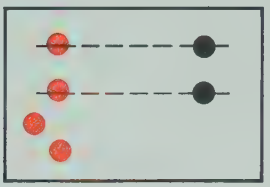
Use counters or checkers if you like. Think of the 2 colors as opposites. Mark out pairs of opposites and solve the equations.

EXAMPLES:

1.

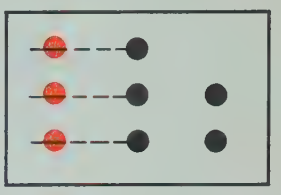


$^{-}2 + 3 =$ 1



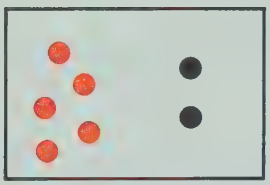
$^{-}4 + 2 =$ $^{-}2$

A



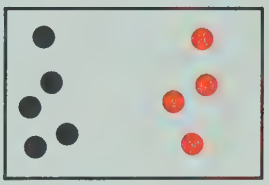
$^{-}3 + 5 =$ 2

B



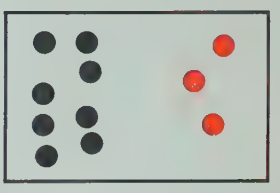
$^{-}5 + 2 =$ $^{-}3$

C



$5 + {}^{-}4 =$ 1

D

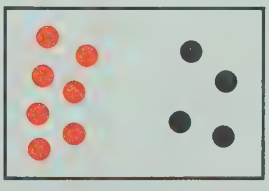


$8 + {}^{-}3 =$ 5

Write the Numbers

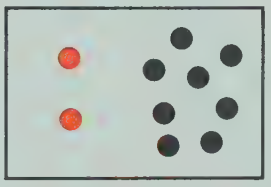
2.

A



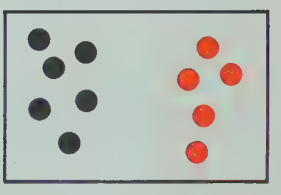
$^{-}7 + 4 = {}^{-}3$

B



$^{-}2 + 8 = 6$

C

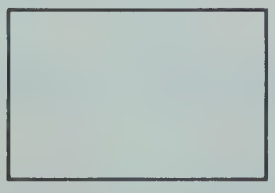


$6 + {}^{-}5 = 1$

Show the Dots

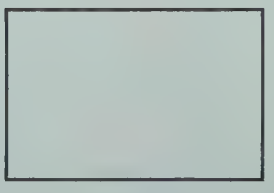
3.

A



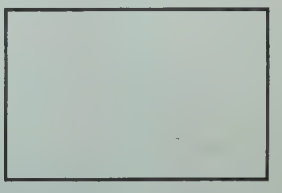
$7 + {}^{-}2 =$ 5

B



$^{-}4 + 5 =$ 1

C



$2 + {}^{-}5 =$ $^{-}3$

Write and solve some equations of your own.

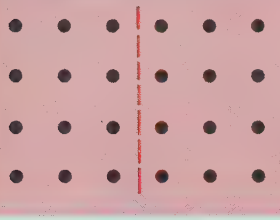
If children have found success in understanding the concepts of negative numbers then addition involving negative numbers should not present much difficulty. By presenting addition through this power skill type method, understanding the concepts involved becomes much easier.

Equations About 24

How many different equations can you write about 24 dots?
If you can, draw lines in each picture to show your thinking.
Use +, −, ×, or ÷. You may use combinations of these if you like.

answers will vary

EXAMPLES:

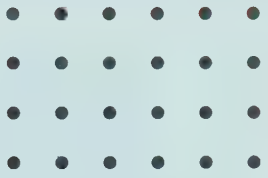


12 + 12 = 24

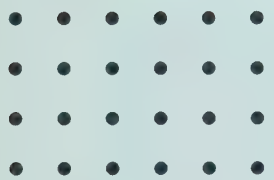
24 ÷ 3 = 8

1.

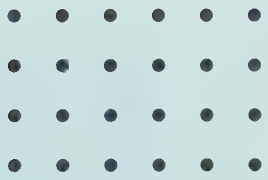
2.



3.

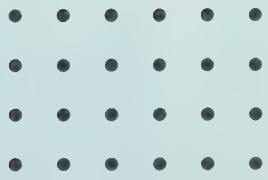


4.



5.

6.

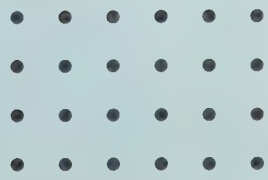


7.

8.

9.

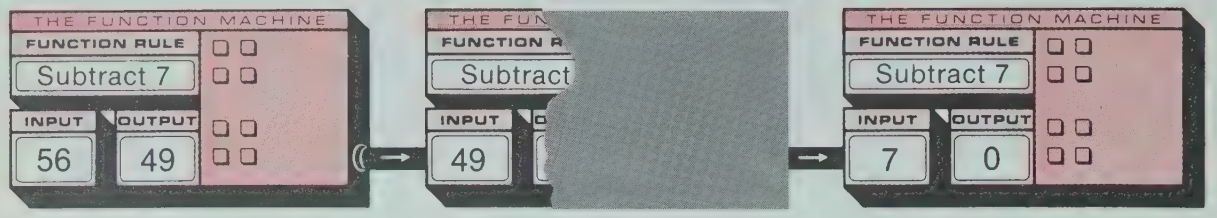
10.



The Function Machine

Each function machine in a row uses the same rule.
The machines are connected so that the output from a machine is the input of the next machine. How many machines are used in each row?

Total Number of Machines

1. 

8

2. 

12

3. 

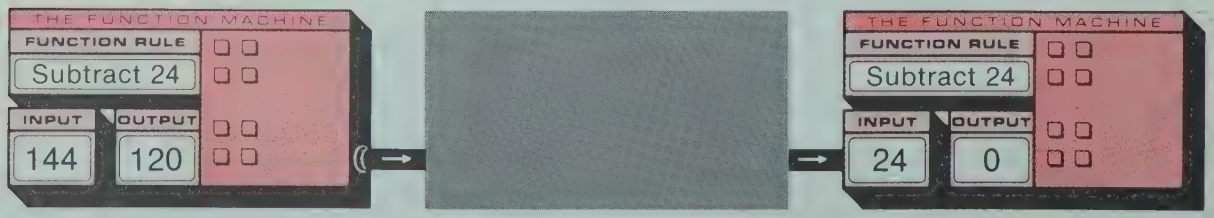
3

4. 

5

5. 

12

6. 

6

The child, by using a process of repeated subtraction can find the quotient. In exercise 1, 7 is subtracted eight times. Thus, there are eight function machines connected together.

Dividing Larger Numbers

Study the example. Subtract the numbers indicated in each problem. Then complete the division equation.

1. 9 4 0 ÷ 4

− 800 ← (200) fours
140
− 120 ← (30) fours
20
− 20 ← (5) fours
0

9 4 0 ÷ 4 = 235

2. 1 2 7 8 ÷ 3

− 1200 ← (400) threes
78
− 60 ← (20) threes
18
− 18 ← (6) threes
0

1 2 7 8 ÷ 3 = 426

3. 3 2 5 8 ÷ 9

− 2700 ← (300) nines
558
− 540 ← (60) nines
18
− 18 ← (2) nines

3 2 5 8 ÷ 9 = 362

---

4. 3 1 6 2 ÷ 6

Use the largest possible of these numbers.

400
(500) sixes
600

− 3000
162
10
(20) sixes
30

− 120
42
5
6 sixes
(7)

− 42
0

3 1 6 2 ÷ 6 = 527

5. 4 5 3 6 ÷ 7

Use the largest possible of these numbers

400
500 sevens
(600)

− 4200
336
(40)
50 sevens
60

− 280
56
7
(8) sevens
9

− 56
0

4 5 3 6 ÷ 7 = 648

6. 1 9 6 8 ÷ 8

Choose the largest possible number for each

− 1600 ← 2 00 eights
368
− 320 ← 4 0 eights
48
− 48 ← 6 eights
0

1 9 6 8 ÷ 8 = 246

Help children to see how the basic facts can be extended to these larger numbers. In the example, to subtract 200 fours we must think, "What is 2 × 4?" The task becomes more difficult when the child must choose the correct multiple to subtract. Again, much emphasis should be placed on

Creating Division Story Problems

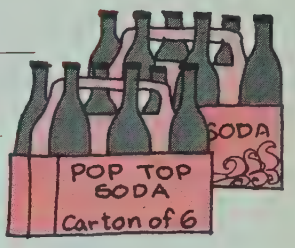
Write a story problem for each picture.
You should be able to solve your problem by solving the equation.

Sample problems

1. Have 144 Baseball Cards. 9 cards from each team. How many teams altogether?

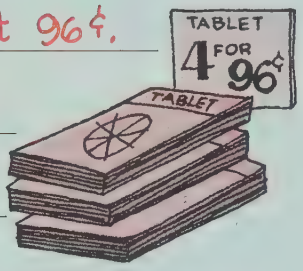
$144 \div 9 = \boxed{16}$

2. There are 126 bottles of soda. One carton has 6 bottles. How many cartons?



$126 \div 6 = \boxed{21}$

3. 4 tablets cost 96¢. How much will one tablet cost?



$96 \div 4 = \boxed{24}$

4. Jan's family lives 425 km from Chicago If they plan to be there in 5 hours, what speed should they travel?

$425 \div 5 = \boxed{85}$

5. Have 192 stamps. Can put 8 stamps on each page. How many pages in the collection?

$192 \div 8 = \boxed{24}$

6. Make one of your own. answers will vary.

$\_\_\_\_ \div \_\_\_\_ = \square$

Number Tricks

Try these.

1.

		Your Choice	Your Choice	Your Choice
Choose a number	3	5	10	21
Add 7	10	12	17	28
Multiply by 2	20	24	34	56
Subtract 4	16	20	30	52
Divide by 2	8	10	15	26
Subtract the number you chose	5	5	5	5

What did you discover? Sample answers The result is always 5.

2. Use the signs $+, -, \times,$ or $\div$ to complete these equations.

A $(4 \div 4) + (4 - 4) = 1$

B $4 \times (4 \div 4) = 4$

C $(4 \div 4) + (4 \div 4) = 2$

D $(4 + 4 + 4) - 4 = 8$

E $(4 + 4 + 4) \div 4 = 3$

F $4 + 4 + 4 + 4 = 16$

3. Use the signs $+, -, \times, \div$ or $=$ to write equations in two different ways.

EXAMPLE: 8 5 3
8 5 3

answer: $8 = 5 + 3$
$8 - 5 = 3$

A $21 \div 7 = 3$
$21 = 7 \times 3$

B $15 - 8 = 7$
$15 = 8 + 7$

C $48 = 6 \times 8$
$48 \div 6 = 8$

D $23 - 15 = 8$
$23 = 15 + 8$

Other Ways to Find Quotients

Find quotients ▢ and ▢. Then use these quotients to find quotient ☐.
Check by multiplying.

1. 20 | 3 | 23

$7\overline{)140}$ $7\overline{)21}$ $7\overline{)161}$

Check: $23 \times 7 = 161$

2. 40 | 2 | 42

$6\overline{)240}$ $6\overline{)12}$ $6\overline{)252}$

$42 \times 6 = 252$

3. 90 | 6 | 96

$4\overline{)360}$ $4\overline{)24}$ $4\overline{)384}$

$96 \times 4 = 384$

4. 80 | 6 | 86

$3\overline{)240}$ $3\overline{)18}$ $3\overline{)258}$

$86 \times 3 = 258$

5. 50 | 3 | 53

$9\overline{)450}$ $9\overline{)27}$ $9\overline{)477}$

$53 \times 9 = 477$

6. 60 | 2 | 62

$8\overline{)480}$ $8\overline{)16}$ $8\overline{)496}$

$62 \times 8 = 496$

7. 70 | 7 | 77

$5\overline{)350}$ $5\overline{)35}$ $5\overline{)385}$

$77 \times 5 = 385$

Story Problems and Equations

Match each story problem with an equation then solve the equation and the problem.

1. A butterfly lived longer than butterflies usually do. It lived 84 days. How many weeks is this? 12

2. There are 584 children in Washington School. On a certain day 8 were absent. How many children were at school? 576

3. There 584 children at Milltown. Only 8 have registered for school. How many have not yet registered? 576

4. You can put 8 softballs in a box. How many boxes will you need to hold 584 softballs? 73

5. If one worker earns 8 dollars an hour, how much will a factory owner have to pay 584 of these workers to work one hour? $4672

A $8 + 576 = 584$

B $8 \times 584 = 4672$

C $584 - 8 = 576$

D $84 \div 7 = 12$

E $584 \div 8 = 73$

Work Space

Exercises of this type are extremely valuable because they focus on one of the most important aspects of story problems, the ability to think through the problem and determine the operation.

Dividing and Remainders

When is the remainder 0?

1. Check ✓ the numbers which give a remainder of 0 when you divide by 2.

A $2\overline{)40}$ ✓ (20; −40; 0)

B $2\overline{)41}$

C $2\overline{)42}$ ✓ (21)

D $2\overline{)43}$

E $2\overline{)44}$ ✓ (22)

F $2\overline{)45}$

G $2\overline{)46}$ ✓ (23)

H $2\overline{)47}$

D $2\overline{)48}$ ✓ (24)

E $2\overline{)49}$

When dividing by 2,
The remainder will be 0 if the number ends in

0, 2, 4, 6, or 8

2. Try some quotients. Then complete the following:

A **When dividing by 10,** the remainder will be 0 if the number ends in 0

B **When dividing by 5,** the remainder will be 0 if the number ends in 0 or 5

You might want to relate this to even and odd numbers. In other words when dividing by 2 the remainder is 0 only when the dividend is an even number.

Sets of Marbles

How many marbles in each bag if each bag has the same number?
Write the number on each bag.

Total Number

1. 

gives **48 marbles**

2. 

gives **17 marbles**

3. 

when doubled gives **32 marbles**

4. 

when doubled gives **28 marbles**

5. 

when divided in half gives **18 marbles**

6. 

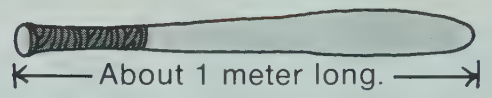
when tripled gives **24 marbles**

Metric Measurement	Which Unit of Length Would You Use?

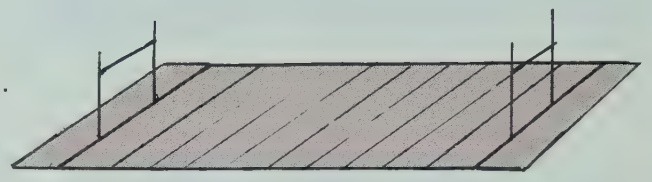
A **centimeter** unit is about this long.

A **millimeter** unit is a tiny unit.
10 millimeters are as long as 1 centimeter.

A **meter** unit is 100 centimeters long.
It is about as long as a baseball bat.



A **kilometer** unit is 1000 meters long.
10 football fields placed end to end
are about 1 kilometer long.



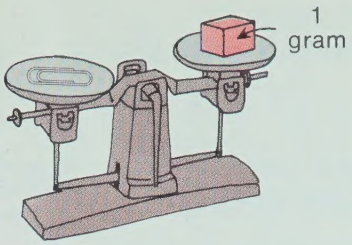
Write the name of the unit which would best be used to measure the following distances.

1. The length of your book centimeter
2. The distance from New York to San Francisco kilometer
3. The thickness of the cover of your book millimeter
4. The height of a person centimeter
5. The height of a building meter
6. The width of your desk centimeter
7. The width of your state kilometer
8. The width of the lead in your lead pencil millimeter

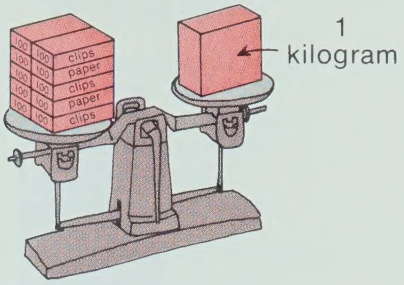
These exercises are intended to be used as on-going activities for metric measurement. They may be expanded and used to reinforce the concepts of the metric system.

Which Unit of Weight Would You Use?

A paper clip weighs about 1 **gram**.



1000 paper clips weigh about 1 **kilogram**.



An 8 or 9 year old child might weigh about 30 **kilograms**.

Something that weighs 1 gram (a paper clip, for example) weighs 1000 **milligrams**. A **milligram** is a tiny unit of weight.

Write the name of the unit above which would best be used in weighing the following:

1. The weight of a cow kilogram

2. The weight of a box of cereal gram

3. The weight of an airplane kilogram

4. The weight of a hair on the leg of a housefly milligram

5. The weight of a nickel gram

6. The weight of a grain of sand milligram

7. The weight of a golf ball gram

8. The weight of a bowling ball kilogram

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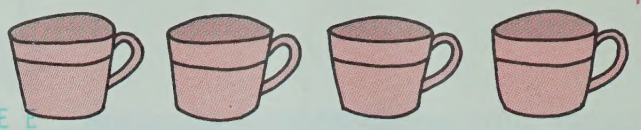
Which Unit of Capacity Would You

A **milliliter** is a small unit of capacity.
A teaspoon holds about 5 **milliliters** of liquid.

A **liter** is a unit 1000 times as much as a milliliter. Four large cups hold almost 1 **liter** of liquid.



A **kiloliter** is as much as 1000 liters. It is a large unit of capacity.

Write the name of the unit above which would best be used in giving these capacities.

1. The amount of medicine in a small medicine bottle milliliter
2. The amount of soda pop in an average bottle liter
3. The amount of water in the Atlantic Ocean kiloliter
4. The amount of gasoline in an automobile tank. liter
5. The amount of gasoline used in the world last year kiloliter
6. The amount of chemical in a 10-centimeter long test tube milliliter
7. The amount of liquid you drink in a day liter

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